Bond Risk Premia and the Macroeconomy

Xiang Fang

The University of Hong Kong

March 2024

Outline

1. Basics of the term structure
2. Bond prices and the macroeconomy
3. Structural models of bond prices
4. Preferred-habitat models

Survey References

- Gurkaynak and Wright (2012 JEL), "Macroeconomics and the Term Structure"
- Duffee (2013 Handbook), "Bond Pricing and the Macroeconomy"
- Piazzesi (2012 Handbook), "Affine Term Structure Models"

1. Basics of the term structure

Building Block: Zero-Coupon Bonds

Zero-coupon bonds: Investors get the face value at maturity and there is no interim coupon payment. Assume no default risk.

- *n*-year maturity bond yield $y_t(n)$

  $$y_t(n) = -\frac{1}{n}\ln(P_t(n))$$

  where $P_t(n)$ is the price of an n-year maturity bond. Continuously compounding is assumed here.

Par Yield Curve

Assume the bond pays coupon semi-annually at stated annual coupon rate of c. The price of such a bond equals

$$P_t(n) = \sum_{i=1}^{2n} \frac{c}{2} d_t(i/2) + d_t(n)$$

where $d_t(i)$ is the price of a zero-coupon bond of maturity i
Par yield $y_t^p(n)$: the coupon rate c such that a security with that maturity would trade at par, i.e. $P_t(n) = 1$

$$1 = \sum_{i=1}^{2n} \frac{y_t^p(n)}{2} d_t(i/2) + d_t(n)$$

Given the zero-coupon rate and semi-annual coupon payment timing, we can solve for the par yield

Duration

- Macaulay duration of a bond: the weighted average of the time that the investor must wait to receive the cash flow on a coupon-bearing bond

$$D = \frac{1}{P_t(n)} \left[\sum_{i=1}^{2n} \frac{i}{2} \frac{c}{2} \frac{1}{(1+y/2)^i} + \frac{n}{(1+y/2)^{2n}} \right]$$

  - Given maturity, higher coupon rate, shorter duration
  - Duration and bond price

$$P_t(n) = \sum_{i=1}^{2n} \frac{c}{2} \frac{1}{(1+y/2)^i} + \frac{1}{(1+y)^n}$$

$$-D \approx \frac{d \ln P_t(n)}{dy_t}$$

Convexity

Notice that

$$d \ln P_t(n) = -Ddy + \frac{1}{2}\kappa(dy)^2$$

where $\kappa = \frac{1}{P}\frac{d^2P}{dy^2}$ is the convexity of the bond.

- Convexity: capital loss from interest rate rise smaller than capital gain from interest rate fall
- Essentially a Jensen's inequality term

Forward Rate

Forward rate: the yield that an investor would require today to make an investment over a specified period in the future, for m years beginning n years hence

$$f_t(n,m) = -\frac{1}{m} \ln \left(\frac{P_t(n+m)}{P_t(n)} \right)$$

$$= \frac{1}{m} \left((n+m) y_t(n+m) - n y_t(n) \right)$$

Take limit $m \to 0$,

$$\lim_{m \to 0} f_t(n,m) = \frac{\partial}{\partial n} \ln P_t(n)$$

Yield can be expressed as

$$y_t(n) = \frac{1}{n} \int_0^n f_t(s,0) ds$$

Yield Curve Estimation

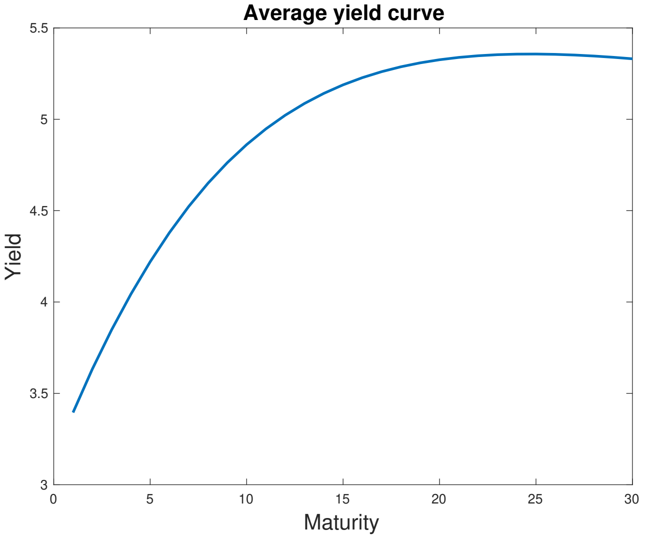
- Zero-coupon bonds are hypothetical - not directly observable
- We don't have securities at all maturities and cannot solve for the implied zero-coupon yields
- Must infer zero-coupon yields across different maturities

Nelson-Sigel-Svensson

$$f_t(n,0) = \beta_0 + \beta_1 \exp(-n/\tau_1) + \beta_2(n/\tau_1)\exp(-n/\tau_1) + \beta_3(n/\tau_2)\exp(-n/\tau_2)$$

- $f_t(0,0) = \beta_0 + \beta_1$, asymptotically to β_0
- Two humps allowed, determined by τ_1 and τ_2
- Parameters to be estimated: $\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2$
- Constructed zero-coupon yield curve regularly updated here
- Start from 1-year, Tbill rate usually does not fit well

The Average Yield Curve



Short Rate Disconnect (Lenel, Piazzesi and Schneider, 2019 JME)

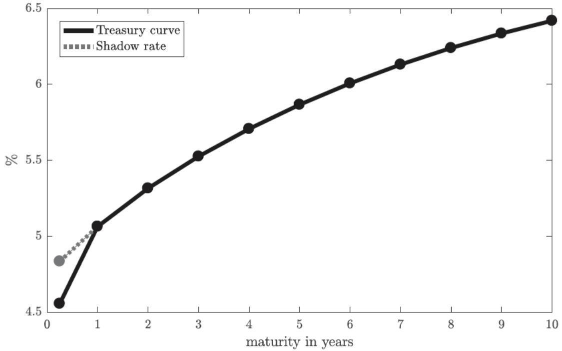
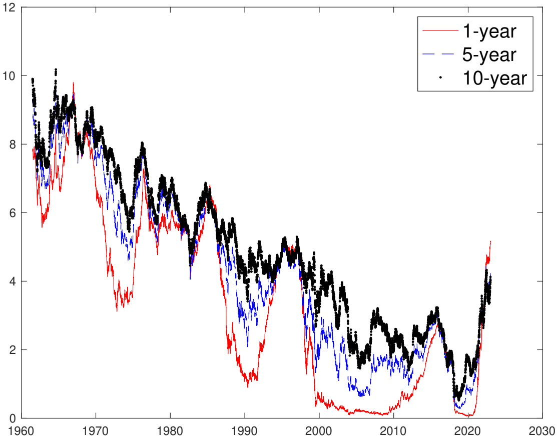


Fig. 1. Average Treasury yield curve in quarterly data as black solid line, 1973–2018. The grey dot is our measure of the shadow rate, the 3-month rate implied by the estimated yield-curve model in Gurkaynak et al. (2007).

Potential reason: the collateral/liquidity use of Treasury bills

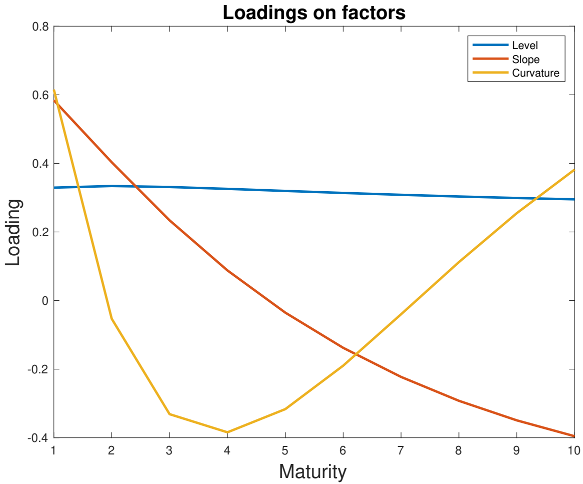
Factor Structure of Yields

Bond yields comove together strongly



Factor Loadings and Factor Interpretations

Factors: the first three PCs of yields



R^2 explained by the three factors cumulatively: 97.85, 99.92, 99.99

Holding Period Return

- Investors do not have to hold the bond to maturity. Instead, they can sell the bond before maturing and then gets exposed to the risk of changing bond price
- The holding period return of an n-period bond for m periods

$$hpr_t(n, m) = \frac{1}{m} \left[\ln P_{t+m}(n-m) - \ln P_t(n) \right]$$

Expectation Hypothesis

- Strong form: long-term yields are equal to the average of expected short-term yields until the maturity

$$y_t(n) = \frac{1}{n}\sum_{i=0}^{n-1} E_t y_{t+i}(1)$$

- Weak form: long-term yields are equal to the average of expeected short term yields until maturity plus a constant term premium
- An average upward-sloping yield curve: If EH holds, it must be in its weak form

Term Premium and Bond Risk Premium

An upward-sloped yield curve = positive average bond risk premium.
Consider an investor holding a $n-j+1$ maturity bond from $t+j-1$ to $t+j$

$$hpr_{t+j}(n-j+1,1) - y_{t+j-1}(1) = \log P_{t+j}(n-j) - \log P_{t+j-1}(n-j+1) - y_{t+j-1}(1)$$

Add up this equation from $j=2$ to n,

$$\sum_{j=2}^{n} hpr_{t+j}(n-j+1,1) - y_{t+j-1}(1) = -\log P_{t+1}(n-1) - \sum_{j=2}^{n} y_{t+j-1}(1)$$

$$= (n-1)y_{t+1}(n-1) - \sum_{j=2}^{n} y_{t+j-1}(1)$$

Take unconditional expectation on both sides,

$$E\sum_{j=2}^{n} hpr_{t+j}(n-j+1,1) - y_{t+j-1}(1) = \frac{1}{n-1}E(y(n-1)-y(1))$$

Rolling over LT bonds with shortening maturities is equivalent to holding a LT bond to maturity.

Three Statements of Expectation Hypothesis

- Long-term yield equals expected future short yields

$$y_t(n) = \frac{1}{n}\sum_{i=0}^{n-1} E_t y_{t+i}(1) + rp$$

- Forward rate equals expected future short rate

$$f_t(n,1) = E_t y_{t+n-1}(1) + rp$$

- Expected holding period return of long-term bond equals the short rate

$$E_t[hpr_{t+1}(n)] = y_t(1) + rp$$

Testing Expectation Hypothesis: Campbell-Shiller

Term premium should predict future short rate changes

$$\sum_{i=1}^{n-1} \frac{n-i}{n}(y_{t+i}(1) - y_{t+i-1}(1)) = \gamma_{n,0} + \gamma_{n,1}(y_t(n) - y_t(1)) + \varepsilon_t$$

Under EH, $\gamma_{n,1} = 1$

n (years)	2	3	4	5
$\gamma_{n,1}$	0.24	0.41	0.56	0.68
$t - stat$	[3.46]	[2.27]	[1.87]	[1.54]
R^2	1.06%	3.42%	7.21%	11.39%

Source: Koijen and van Nieurburgh's lectures notes, section 8

EH fails: yield spread does forecast future short rate changes but not strong enough

Testing Expectation Hypothesis: Campbell-Shiller

Bond excess return should not be predictable by yields

$$\frac{1}{n}\sum_{i=0}^{n-1} r_{t+i-1}(n-i) - y_{t+i}(1) = \gamma_{n,0} + \gamma_{n,1}(y_t(n) - y_t(1)) + \varepsilon_t$$

Under EH, $\gamma_{n,1} = 0$

n (years)	2	3	4	5
$\gamma_{n,1}$	0.76	0.59	0.44	0.32
$t-stat$	[3.46]	[2.27]	[1.87]	[1.54]
R^2	9.50%	6.86%	4.59%	2.87%

Source: Koijen and van Nieurburgh's lectures notes, section 8

EH fails: bond excess returns are predictable using yield spread. A higher yield spread is associated with a higher future risk premium

Testing Expectation Hypothesis: Fama-Bliss

The forward rate should predict future short rate change

$$y_{t+n-1}(1) - y_t(1) = a_{n,0} + a_{n,1}(f_t(n) - y_t(1)) + \varepsilon_{t+n-1}$$

Under EH, $a_{n,1} = 1$

n (years)	2	3	4	5
$\alpha_{n,1}$	0.24	0.54	0.72	0.78
$t - stat$	[-3.46]	[-1.57]	[-1.34]	[-1.18]
R^2	1.06%	5.50%	11.50%	14.66%

Source: Koijen and van Nieurburgh's lectures notes, section 8

EH fails: future yield changes are too small compared to changes in forward rate

Testing Expectation Hypothesis: Fama-Bliss

Bond excess return should not be predictable by forward rate

$$r_{t+1}(n) - y_t(1) = \gamma_{n,0} + \gamma_{n,1}(f_t(n) - y_t(1)) + \varepsilon_{t+1}$$

Under EH, $\gamma_{n,1} = 0$.

n (years)	2	3	4	5
$\gamma_{n,1}$	0.76	1.00	1.27	1.06
$t - stat$	[3.46]	[3.59]	[4.03]	[3.00]
R^2	9.50%	10.63%	13.58%	7.88%

Source: Koijen and van Nieurburgh's lectures notes, section 8

EH fails: bond excess returns are predictable using forward rates

Bond Return Predictability: Cochrane-Piazzesi (2005 AER)

Improve bond return predictability using all forward rates?

1. Regress average excess return onto short rate and forward rates

$$\frac{1}{4}\sum_{n=2}^{5} rx_{t+1}^{(n)} = \gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + \ldots + \gamma_5 f_t^{(5)} + \varepsilon_{t+1}$$

2. Use the fitted value as a single predictor

$$rx_{t+1}^{(n)} = b_n(\hat{\gamma}' f_t) + \varepsilon_{t+1}^{(n)}$$

Cochrane-Piazzesi Factor

TABLE 1—ESTIMATES OF THE SINGLE-FACTOR MODEL

A. Estimates of the return-forecasting factor, $\overline{rx}_{t+1} = \gamma^\top \mathbf{f}_t + \bar{\varepsilon}_{t+1}$

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5	R^2	$\chi^2(5)$
OLS estimates	−3.24	−2.14	0.81	3.00	0.80	−2.08	0.35	
			Asymptotic (Large T) distributions					
HH, 12 lags	(1.45)	(0.36)	(0.74)	(0.50)	(0.45)	(0.34)		811.3
NW, 18 lags	(1.31)	(0.34)	(0.69)	(0.55)	(0.46)	(0.41)		105.5
Simplified HH	(1.80)	(0.59)	(1.04)	(0.78)	(0.62)	(0.55)		42.4
No overlap	(1.83)	(0.84)	(1.69)	(1.69)	(1.21)	(1.06)		22.6
			Small-sample (Small T) distributions					
12 lag VAR	(1.72)	(0.60)	(1.00)	(0.80)	(0.60)	(0.58)	[0.22, 0.56]	40.2
Cointegrated VAR	(1.88)	(0.63)	(1.05)	(0.80)	(0.60)	(0.58)	[0.18, 0.51]	38.1
Exp. Hypo.							[0.00, 0.17]	

B. Individual-bond regressions

	Restricted, $rx^{(n)}_{t+1} = b_n(\gamma^\top \mathbf{f}_t) + \varepsilon^{(n)}_{t+1}$					Unrestricted, $rx^{(n)}_{t+1} = \beta_n \mathbf{f}_t + \varepsilon^{(n)}_{t+1}$			
n	b_n	Large T	Small T	R^2	Small T	R^2	EH	Level R^2	$\chi^2(5)$
2	0.47	(0.03)	(0.02)	0.31	[0.18, 0.52]	0.32	[0, 0.17]	0.36	121.8
3	0.87	(0.02)	(0.02)	0.34	[0.21, 0.54]	0.34	[0, 0.17]	0.36	113.8
4	1.24	(0.01)	(0.02)	0.37	[0.24, 0.57]	0.37	[0, 0.17]	0.39	115.7
5	1.43	(0.04)	(0.03)	0.34	[0.21, 0.55]	0.35	[0, 0.17]	0.36	88.2

Notes: The 10-percent, 5-percent and 1-percent critical values for a $\chi^2(5)$ are 9.2, 11.1, and 15.1 respectively. All p-values are less than 0.005. Standard errors in parentheses "( )", 95-percent confidence intervals for R^2 in square brackets "[ ]". Monthly observations of annual returns, 1964–2003.

Summary: What Do We Know About Yield Curve?

- Strong factor structure, 3 principal components explain basically all movements
- Upward-sloping yield curve
- EH fails and risk premium is time-varying. Bond excess returns can be predicted by a combination of forward rates (Cochrane-Piazzesi factor)
- How are yields related to the macroeconomy?
- What are the restrictions imposed on yield dynamics?

Why Do We Care About Bond Yields?

- Forecast future paths of the macroeconomy using information in bond yields
- Monetary policy moves short rate but what affects the real economy is the long rate. Understanding the term structure helps understand the monetary policy transmission
- Debt policy: government decides about the maturity, whose effect depends on the determination of bond yields
- Derivative pricing and hedging: traders need to know how to price derivatives depending on the state of economy

2. Bond prices and the macroeconomy

Main Questions

- Macro: What do bond yields tell us about the macroeconomy?
- Finance: What are the macro risks embedded in bond yields?
  - How to bond yields respond to macroeconomic shocks?
  - How can macroeconomic factors help forecast bond yields?

Simple VAR

$$X_{t+1} = \mu + KX_t + \Sigma\varepsilon_{t+1}, \varepsilon_{t+1} \sim MVN(0, I)$$

- To obtain IRFs, we need to impose some restrictions on Σ
- Limitations
  - Yields are not that much spanned by macroeconomic factors
  - Lack of no-arbitrage restrictions
  - Only able to forecast yields included in the VAR

State-Space Model

- p-factor state dynamics

$$x_{t+1} = \mu + Kx_t + \Sigma\varepsilon_{t+1}, \varepsilon_{t+1} \sim MVN(0, I)$$

- Measurement equation

$$\tilde{z}_t = \mathbf{A} + \mathbf{B}x_t + \eta_t, \eta_t \sim MVN(0, \Omega)$$

- Interpretation
  - x_t captures common variation in the observables
  - η_t the idiosyncratic deviations (cross-sectional errors)
- Observables: bond yields, inflation, real consumption growth
- m observable yields can be used to invert m latent factors

How to estimate this model? See here (page 85-168)

No-Arbitrage: SDF and Short Rates

Bond prices should satisfy no-arbitrage relations

- Denote M_{t+1} the real SDF, π_{t+1} the inflation rate, the nominal SDF is thus

$$M^{\$}_{t+1} = M_{t+1} \exp(-\pi_{t+1})$$

- Real interest rate

$$r_t = -E_t m_{t+1} - \frac{1}{2} var_t(m_{t+1})$$

- Nominal interest rate

$$r^{\$}_t = -E_t m^{\$}_{t+1} - \frac{1}{2} var_t(m^{\$}_{t+1})$$

SDF Dynamics, Short Rate and Price of Risk: Real

- Real SDF

$$m_{t+1} = -r_t - \frac{1}{2}\Lambda_t^{'}\Lambda_t - \Lambda_t^{'}\varepsilon_{t+1}$$

- Short rate

$$r_t = -\delta_0 + \delta_1^{'} x_t$$

- Price of risk

$$\Lambda_t = \Sigma^{-1}(\lambda_0 + \lambda_1 x_t)$$

SDF Dynamics, Short Rate and Price of Risk: Nominal

- Suppose inflation follows

$$\tilde{\pi}_t = A_\pi + B_\pi^{'} x_t + \eta_{\pi,t}$$

- Nominal SDF

$$m_{t+1}^{\$} = -r_t^{\$} - \frac{1}{2}\Lambda_t^{\$'}\Lambda_t^{\$} - \frac{1}{2}\Omega_\pi - \Lambda_t^{\$'}\varepsilon_{t+1} - \eta_{\pi,t+1}$$

- Short rate

$$r_t^{\$} = \delta_0^{\$} + \delta_1^{\$'} x_t$$

  where

$$\delta_0^{\$} = \delta_0 + A_\pi + B_\pi^{'}(\mu - \lambda_0) - \frac{1}{2}B_\pi^{'}\Sigma\Sigma^{'}B_\pi - \frac{1}{2}\Omega_\pi$$

$$\delta_1^{\$'} = \delta_1^{'} + B_\pi^{'}(K - \lambda_0)$$

- Price of risk

$$\Lambda_t^{\$} = \Sigma^{-1}(\lambda_0^{\$} + \lambda_1^{\$} x_t)$$

  where

$$\lambda_0^{\$} = \lambda_0 + \Sigma\Sigma^{'}B_\pi, \lambda_1^{\$} = \lambda_1$$

Bond Prices: Recursive Solution

$$P_t^{(n)} = \exp(A(n) + B(n)' x_t)$$

Plug in the asset pricing equation $E_t M_{t+1} P_{t+1}^{(n-1)} = P_t^{(n)}$, we solve for $A(n)$ and $B(n)$ and derive the recursive solution

$$B(n)' = B(n-1)'(K - \lambda_1) - \delta_1'$$

$$A(n)' = A(n-1) + B(n-1)'(\mu - \lambda_0) - \delta_0 - \frac{1}{2} B(n-1)' \Sigma \Sigma' B(n-1)$$

The log expected return of an n-period bond from t to $t+1$

$$\log E_t(R_{t+1}^{(n)}) = r_t + B(n-1)'(\lambda_0 + \lambda_1 x_t)$$

No-arbitrage imposes restrictions on VAR coefficients **A, B**

Bond Prices under the Q Measure

We can write out the dynamics of x_t under the Q measure

$$x_{t+1} = \mu^q + K^q x_t + \Sigma \varepsilon^q_{t+1}$$

where $\mu^q = \mu - \lambda_0, K^q = K - \lambda_1$. Then bond prices can be written as

$$P^{(n)}_t = e^{-r_t} E^Q_t \left(P^{(n-1)}_{t+1} \right)$$

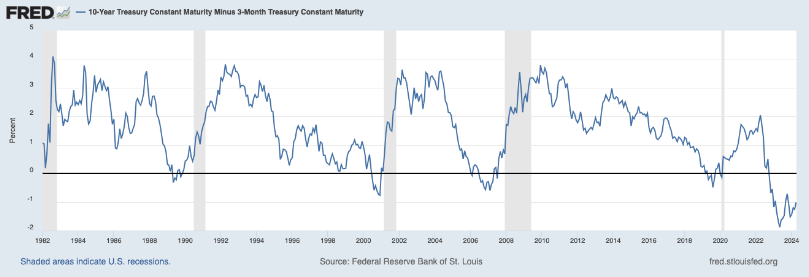
We can also solve for $A(n)$ and $B(n)$ recursively as

$$B(n)' = B(n-1)' K^q - \delta_1'$$

$$A(n)' = A(n-1) + B(n-1)' \mu^q - \delta_0 - \frac{1}{2} B(n-1)' \Sigma \Sigma' B(n-1)$$

Yield Curve Slope and the Macroeconomy

The shape of the term structure of interest rates is an important indicator of the macroeconomy, especially the slope



Negative term premia predicts recessions pre-2020

Two Explanations

- A flat yield curve is because investors anticipate the recession and thus anticipate lower future interest rates
- The flat yield curve *causes* the recession
  - Minoiu, Schneider and Wei (2023): supressed term premia lowers bank profits and decreases credit supply

Specifying x_t

- Using latent factors, say, level, slope and curvature
  - Good fit but unclear economic interpretations
- Using factors extracted from macroeconomic variables, for example, expected growth and expected inflation
  - Clear economic interpretation but usually poor fit
- A combination of the two (Ang and Piazzesi, 2003 JME)
  - Better explain yield dynamics with macro factors

Model Specification

- State factor F_t follows VAR(p), $p = 12$

$$F_t = \mu + \Phi_1 F_{t-1} + ... + \Phi_p F_{t-p} + \Sigma u_t$$

  - $F_t = (f_t^{o'}, f_t^{u'})$ where f_t^o are observable macro factors ($K_1 = 2$) and f_t^u are unobservable latent variables ($K_2 = 3$)
  - Companion form

$$x_t = \mu + \Phi x_{t-1} + \Sigma \varepsilon_t$$

    where

$$x_t = (x_t^{o'}, x_t^{u'})', x_t^o = (f_t^{o'}, f_{t-1}^{o'}, ..., f_{t-p-1}^{o'}), x_t^u = f_t^u$$

- Short rate $r_t = \delta_0 + \delta_1^{'} x_t = \delta_0 + \delta_{11}^{'} x_t^o + \delta_{12} x_t^u$
  - δ_1 unconstrained, can depend on lagged macro variables
- Pricing kernel and the price of risk

$$m_{t+1}^{\$} = -r_t^{\$} - \frac{1}{2}\Lambda_t^{'}\Lambda_t - \Lambda_t^{'}\varepsilon_t$$

$$\Lambda_t = \lambda_0 + \lambda_1 x_t$$

Restricted Model

- Unobservable factors

$$x_t^u = \rho x_{t-1}^u + u_t^u, u_t^u \sim iid\ N(0, I)$$

  Normalizations

  - Zero mean, ρ is lower-triangular, identity covariance matrix - general identified representation

- Independence between macro factors and latent factors
- The covariance matrix of f_t^o is lower diagonal
  - Limitation: monetary policy has no real effect
- Macro variables: inflation and real activity, both PCs

Estimation

- Observable: $(y_t', x_t^{o'})$
- By no-arbitrage, y_t are affine functions of x_t, so we can infer latent variables from observed yields
- Assume three yields are measured without error so that the three unobserved latent factors are exactly identified

Estimate: Macro Model

Restricting the short rate only depends on *contemporaneous* factors

Table 6
Macro model estimates

Companion form Φ for latent factors

0.9915 (0.0042)	0.0000	0.0000
0.0000	0.9392 (0.0122)	0.0000
0.0000	0.0125 (0.0146)	0.7728 (0.0217)

Short rate parameters δ_1 for latent factors ($\times$100)

Unobs 1	Unobs 2	Unobs 3
0.0138 (0.0021)	−0.0487 (0.0007)	0.0190 (0.0022)

Prices of risk λ_0 and λ_1

	λ_0	λ_1 matrix				
		Inflation	Real activity	Unobs 1	Unobs 2	Unobs 3
Inflation	0.0000	−0.4263 (0.1331)	0.1616 (0.0146)	0.0000	0.0000	0.0000
Real activity	0.0000	1.9322 (0.3893)	−0.1015 (0.0329)	0.0000	0.0000	0.0000
Unobs 1	−0.0039 (0.0003)	0.0000	0.0000	−0.0047 (0.0043)	0.0000	0.0000
Unobs 2	0.0000	0.0000	0.0000	0.0459 (0.0055)	0.0000	−0.2921 (0.0205)
Unobs 3	0.0000	0.0000	0.0000	−0.0351 (0.0087)	0.0000	0.1995 (0.0283)

Measurement error ($\times$100)

3 month	36 month
0.0207 (0.0003)	0.0091 (0.0002)

The table reports parameter estimates and standard errors in parenthesis for the Macro Model with the short rate equation specified with only current inflation and current real activity, as reported in Panel A of Table 4. The short rate equation is given by $r_t = \delta_0 + \delta_1' X_t$, where δ_1 only picks up current inflation, current real activity and the latent factors. The dynamics of inflation and real activity are given by a 12 lag VAR (not reported). The model is $X_t = \Phi X_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim N(0, I)$. X_t contains 12 lags of inflation and real activity and three latent variables, which are independent at all lags to the macro variables. In a pre-estimation we find the inflation and real activity VAR(12), and the coefficients on inflation and real activity in the short rate equation. The coefficient δ_0 is set to the sample unconditional mean of the short rate, 0.0513/12. Market prices of risk $\lambda_t = \lambda_0 + \lambda_1 X_t$ are restricted to be block diagonal. The sample period is 1952:06 to 2000:12.

Estimate: Macro Model with Lags

Table 7
Macro lag model estimates

Companion form Φ for latent factors

0.9922	0.0000	0.0000
(0.0039)		
0.0000	0.9431	0.0000
	(0.0118)	
0.0000	−0.0189	0.8210
	(0.0135)	(0.0216)

Short rate parameters δ_1 for latent factors ($\times 100$)

Unobs 1	Unobs 2	Unobs 3
0.0130	−0.0438	0.0256
(0.0020)	(0.0010)	(0.0025)

Prices of risk λ_0 and λ_1

	λ_0	λ_1 matrix				
		Inflation	Real activity	Unobs 1	Unobs 2	Unobs 3
Inflation	0.0000	0.8442	−0.0017	0.0000	0.0000	0.0000
		(0.2397)	(0.0582)			
Real activity	0.0000	1.1209	0.2102	0.0000	0.0000	0.0000
		(0.1375)	(0.0275)			
Unobs 1	−0.0050	0.0000	0.0000	−0.0048	0.0000	0.0000
	(0.0003)			(0.0040)		
Unobs 2	0.0000	0.0000	0.0000	0.0483	0.0000	−0.2713
				(0.0068)		(0.0195)
Unobs 3	0.0000	0.0000	0.0000	−0.0248	0.0000	0.1624
				(0.0078)		(0.0292)

Measurement error ($\times 100$)

3 month	36 month
0.0251	0.0107
(0.0005)	(0.0003)

The table reports parameter estimates and standard errors in parenthesis for the Macro Lag Model with the short rate equation specified with 12 lags of inflation and current real activity, as reported in Panel B of Table 4. The short rate equation is given by $r_t = \delta_0 + \delta_1' X_t$, where δ_1 only picks up 12 lags of inflation and real activity and the latent factors. The dynamics of inflation and real activity are given by a 12 lag VAR (not reported). The model is $X_t = \Phi X_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim N(0, I)$. X_t contains 12 lags of inflation and real activity and three latent variables, which are independent at all lags to the macro variables. In a pre-estimation we find the inflation and real activity VAR(12), and the coefficients on inflation and real activity in the short rate equation. The coefficient δ_0 is set to the sample unconditional mean of the short rate, 0.0513/12. Market prices of risk $\lambda_t = \lambda_0 + \lambda_1 X_t$ are restricted to be block diagonal. The sample period is 1952:06 to 2000:12.

- The price of inflation and real activity risk differs in the "macro model" and "macro model with lags"
- Mainly due to different specifications of the short rate

Yield Impulse Responses

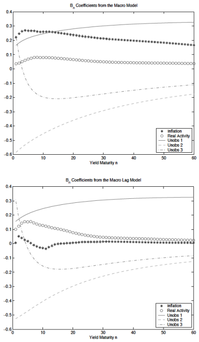


Fig. 4. B_n yield weights for the macro and macro lag model. The figure displays B_n yield weights as a function of maturity n for the Macro (Macro Lag) model in the top (bottom) plot. The plots show only the B_n yield weights corresponding to contemporaneous state variables in each system. The weights are scaled to correspond to one standard deviation movements in the factors and are annualized by multiplying by 1200.

- Three latent factors: level, slope and curvature
- Inflation and real activity: differ across models, mainly respond on the short end
- Due to different estimates of δ_{11} and λ_1

Impulse Respones: Comparison

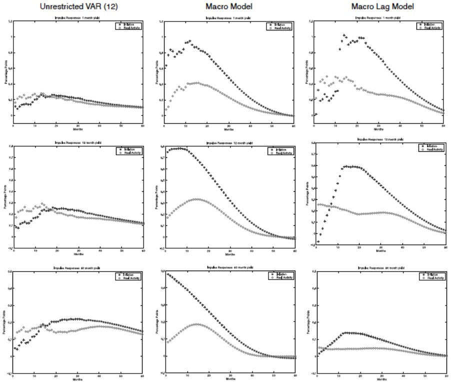


Fig. 5. Impulse response functions. Impulse Responses (IR's) for 1 month (top row), 12 month (middle row) and 60 month (bottom row) yields. The first column presents IR's from an unrestricted VAR(12) fitted to macro variables and yields ; the middle column presents IR's from the Macro model; and the last column presents IR's from the Macro Lag model. The IR's from the latent factors are drawn as lines, while the IR's from inflation (real activity) are drawn as stars (circles). All IR's are from a one standard deviation shock.

- Imposing no-arbitrage leads to much larger impulse responses to both inflation and real activity shocks

Variance Decomposition of Forecast

Table 8
Proportion of variance explained by macro factors

	Forecast horizon h			
	1 mth	12 mth	60 mth	∞
Macro model				
Short end	50%	78%	85%	83%
Middle	67%	79%	78%	73%
Long end	61%	63%	48%	38%
Macro lag model				
Short end	11%	57%	87%	85%
Middle	23%	52%	71%	64%
Long end	2%	8%	11%	7%

We list the contribution of the macro factors to the h-step ahead forecast variance of the 1 month yield (short end), 12 month yield (middle) and 60 month yield (long end) for the Macro and Macro Lag Models. These are the sum of the variance decompositions from the macro factors in Table 9.

- Macro variables explain a large fraction of forecast variance at different forecast horizons, especially the short and middle end
- Additional result (not shown but in the paper): inflation explains a much larger fraction than real activity

Out-of-Sample Forecast

Table 10
Forecast comparisons

Yield (mths)	RW	Unconstrained VARs		VARs with cross-equation restrictions		
		VAR Yields Only	VAR with Macro	Yields Only	Macro model	Macro lag model
RMSE criteria						
1	0.3160	0.3905	0.3990	0.3012	**0.2889**	0.3906
3	**0.1523**	0.2495	0.2540	0.1860	0.2167	0.2876
12	0.1991	0.2776	0.2722	0.1914	**0.1851**	0.2274
36	0.2493	0.3730	0.3644	0.2489	**0.2092**	0.2665
60	0.2546	0.3793	0.3725	0.2497	**0.2333**	0.2530
MAD criteria						
1	0.2252	0.3076	0.3242	0.2155	**0.2039**	0.2981
3	**0.1159**	0.1987	0.2056	0.1442	0.1693	0.2344
12	0.1639	0.2176	0.2204	0.1616	**0.1559**	0.1870
36	0.1997	0.2991	0.2924	0.1974	**0.1604**	0.2111
60	0.2054	0.2957	0.2930	0.2017	**0.1883**	0.2064

We forecast over the last 60 months (the out-sample) of our sample and record the root mean square error (RMSE) and the mean absolute deviation (MAD) of the forecast versus the actual values. Lower RMSE and MAD values denote better forecasts, with the best statistics highlighted in bold. Forecasts are 1-step ahead. We first estimate models on the in-sample, and update the estimations at each observation in the out-sample. RW denotes a random walk forecast, VAR Yields Only denotes a VAR(12) only with 5 yields, VAR with Macro denotes a VAR(12) fitted to the macro factors and all 5 yields, Yields-Only denotes the 3 factor latent variable model without macro variables, the Macro model has only contemporaneous inflation and real activity in the short rate equation, and the Macro Lag model has contemporaneous and 12 lags of inflation and real activity in the short rate equation. The first three of these models are thus unconstrained estimations, while the last three impose the cross-equation restrictions derived from the absence of arbitrage.

- Including macro factors greatly improves forecasting performance

Comparison of Factors

Table 11
Comparison of Yields-Only and macro factors

Dependent variable	Independent variables					
	Inflation	Real activity	Unobs 1	Unobs 2	Unobs 3	Adj R^2
Panel A: Regressions on macro factors						
Unobs 1	0.4625	−0.0726				0.2180
"level"	(0.0735)	(0.0860)				
Unobs 2	−0.6707	−0.1890				0.4902
"spread"	(0.0716)	(0.0611)				
Unobs 3	0.0498	−0.1794				0.0343
"curvature"	(0.0629)	(0.0714)				
Panel B: Regressions on factors from macro model						
Unobs 1	0.1118	0.0307	0.9507	−0.0174	0.0038	0.9971
	(0.0054)	(0.0056)	(0.0055)	(0.0056)	(0.0047)	
Unobs 2	−0.9364	−0.1026	0.0199	0.7624	0.0279	0.9981
	(0.0037)	(0.0037)	(0.0042)	(0.0032)	(0.0029)	
Unobs 3	0.0427	−0.1238	0.1656	−0.1455	0.9071	0.9256
	(0.0262)	(0.0260)	(0.0289)	(0.0241)	(0.0233)	
Panel C: Regressions on factors from macro lag model						
Unobs 1	−0.0580	−0.0207	1.0248	0.0035	0.0058	0.9979
	(0.0049)	(0.0040)	(0.0044)	(0.0047)	(0.0036)	
Unobs 2	−0.7069	−0.1132	−0.2955	0.5700	0.1306	0.8715
	(0.0393)	(0.0313)	(0.0356)	(0.0376)	(0.0315)	
Unobs 3	0.1112	−0.0081	0.2059	0.0228	0.8119	0.7470
	(0.0458)	(0.0386)	(0.0507)	(0.0365)	(0.0424)	

Regressions of the latent factors from the Yields-Only model with only latent factors (dependent variables) onto the macro factors and latent factors from the Macro and Macro Lag model (independent variables). All factors are normalized, and standard errors, produced using 3 Newey–West (1987) lags, are in parentheses. Panel A lists coefficients from a regression of the Yields-Only latent factors onto only macro factors. Panel B lists coefficients from a regression of Yields-Only latent factors on the macro and latent factors from the Macro model with only contemporaneous inflation and real activity in the short rate equation. Panel C lists coefficients from a regression of Yields-Only latent factors on the macro and latent factors from the Macro Lag model with contemporaneous inflation and real activity and 11 lags of inflation and real activity in the short rate equation.

- Macro factors have substantial explanatory power on the latent factors, especially the level and slope

Takeaway: Ang and Piazzesi (2003)

- The pioneering work combining latent factors and macro factors in affine term structure models
- Including macro factors in a no-arbitrage model is useful in understanding yield impulse responses and improving forecast
- The traditional latent factors, especially level and slope, are significantly associated with macroeconomic factors

Estimating Affine Term Structure Models

- Maximum likelihood (traditional method)
  - Sample code on Gregory Duffee's website
- Linear regression (Hamilton and Wu, 2012 JoE)
  - Sample code on Cynthia Wu's website
- A simpler method (Joslin, Singleton and Zhu, 2011 JFE)
  - Sample code on Scott Joslin's website
- Linear regression using bond returns instead of yields (Adrian, Crump and Moench, 2013 JFE)

Treasury Risk Premium: Cieslak and Pavola (2015, RFS)

- What drives the risk premium in Treasury bonds?
- Idea: risk premium = yields - inflation expectation - short rate expectation
  - Inflation expectation: trend inflation
  - Short-rate *cycle*: the short rate that is orthogonal to trend inflation

An Illustrative Affine Model

- Three state variables: trend inflation τ_t, real factor r_t, price of risk factor x_t
  - Trend inflation

$$\tau_t = \mu_\tau + \phi_\tau \tau_{t-1} + \sigma_\tau \varepsilon_t^\tau$$

    Realized inflation $\pi_{t+1} = \tau_t + \varepsilon_{t+1}^\pi$, then τ_t is the expected inflation

- Short rate

$$y_t^{(1)} = \delta_0 + \delta_\tau \tau_t + \delta_r r_t$$

- Real factor

$$r_t = \mu_r + \phi_r r_{t-1} + \sigma_r \varepsilon_t^r$$

  Interpreted as variation in the real short rate *independent of* the trend inflation, $\delta_r r_t$ is the short rate cycle

- The price of risk factor

$$x_t = \mu_x + \phi_x x_{t-1} + \sigma_x \varepsilon_t^x$$

- Assume x shocks are not priced but x affects the price of risk for τ and r

Solution

- Yield can be expressed as an affine function of τ, r, x

$$y_t^{(n)} = A_n + B_n^\tau \tau_t + B_n^r r_t + B_n^x x_t$$

- Define "cycle" at each maturity (orthogonal to trend inflation) as

$$c_t^{(n)} = B_n^r r_t + B_n^x x_t$$

  For $n = 1$, $c_t^{(1)} = \delta_r r_t$, not load on x
- x_t drives the Treasury risk premium, and the relative contribution of x_t to the cycles' variance increases with maturity
- Empirical goal: extract x_t from the cycles of yields

Trend Inflation Measurement

$$\tau_t^{CPI} = (1 - v) \sum_{i=0}^{t-1} v^i \pi_{t-i}$$

where π_t is the year-over-year inflation

- Reflect sluggish response of inflation expectation
- $v = 0.987$, calibrated to survey data
- Use core CPI to construct trend inflation
- Interest rate responds to trend inflation more than one for one

Interest Rate Cycles

- Project yields at each maturity onto trend inflation

$$y_t^{(n)} = a_n + b_n^\tau \tau_t^{CPI} + \varepsilon_t^{(n)}$$

Construct the interest rate cycle (maturity-specific) as

$$c_t^{(n)} = y_t^{(n)} - \hat{a}_n - \hat{b}_n^\tau \tau_t^{CPI}$$

A. Regressions of yields on τ_t^{CPI}: $y_t^{(n)} = a_n + b_n^\tau \tau_t^{CPI} + \varepsilon_t$

	$y_t^{(1)}$	$y_t^{(2)}$	$y_t^{(5)}$	$y_t^{(7)}$	$y_t^{(10)}$	$y_t^{(15)}$	$y_t^{(20)}$
$a_n \times 100$	−0.35	−0.12	0.68	1.09	1.43	1.97	2.51
	(−0.45)	(−0.17)	(1.47)	(2.87)	(4.66)	(7.44)	(8.91)
b_n^τ	1.43	1.44	1.37	1.32	1.28	1.20	1.13
	(8.64)	(10.31)	(13.06)	(14.58)	(15.98)	(16.26)	(14.93)
$\bar{R}^2$	0.71	0.77	0.84	0.86	0.88	0.89	0.86

Predictive Regression

Predicting average excess returns $\bar{rx}_{t+1}$

Table 2
Predictive regressions

A. Predictive regressions

Regressors →	Yields only (1)	Yields+τ^{CPI} (2)	$\bar{y}_t, y_t^{(1)}$ (3)	$\bar{y}_t, y_t^{(1)}, \tau_t^{CPI}$ (4)	$\bar{c}_t, c_t^{(1)}$ (5)
Regression coefficients					
$y^{(1)}$ or $c^{(1)}$	−1.13	−1.09	−0.42	−0.61	−0.61
	(−1.87)	(−1.64)	(−2.48)	(−3.70)	(−3.67)
$y^{(2)}$ or $c^{(2)}$	0.73	1.06	–	–	–
	(0.62)	(0.81)	–	–	–
$y^{(5)}$ or $c^{(5)}$	0.83	−0.71	–	–	–
	(0.99)	(−0.10)	–	–	–
$y^{(7)}$ or $c^{(7)}$	0.40	0.51	–	–	–
	(0.15)	(0.32)	–	–	–
$y^{(10)}$ or $c^{(10)}$	−1.15	0.84	–	–	–
	(−1.69)	(0.43)	–	–	–
$y^{(20)}$ or $c^{(20)}$	0.37	0.21	–	–	–
	(0.94)	(0.49)	–	–	–
τ^{CPI}	–	−1.02	–	−1.01	–
	–	(−4.30)	–	(−4.65)	–
$\bar{y}$ or $\bar{c}$	–	–	0.54	1.45	1.45
	–	–	(2.47)	(5.03)	(5.03)
Regression statistics					
$\bar{R}^2$	0.24	0.54	0.18	0.53	0.53
Wald test	12.34	34.86	6.46	28.61	25.34
pval	0.05	0.00	0.04	0.00	0.00
Rel.prob. (BIC)	0	3e-4	0	0.57	1.00

B. Distribution of predictive $\bar{R}^2$ under EH, $T=470$ months

	$\phi_r=0.75$			$\phi_\tau=0.975$		
	$\phi_\tau=0.8$	$\phi_\tau=0.975$	$\phi_\tau=0.999$	$\phi_r=0.6$	$\phi_r=0.75$	$\phi_r=0.9$
P5	0.00	0.01	0.01	0.01	0.01	0.01
P95	0.19	0.23	0.20	0.22	0.22	0.23

In panel A, the LHS variable is a duration-standardized excess bond return averaged across maturities, $\overline{rx}_{t+1}$. Columns (1) through (5) use different regressors: (1) six yields; (2) same yields as in (1) plus trend inflation τ_t^{CPI}; (3) two yield variables: $y_t^{(1)}$ and $\bar{y}_t$; (4) $y_t^{(1)}$ and $\bar{y}_t$ plus τ_t^{CPI}; (5) two cycle variables: $c_t^{(1)}$ and the average cycle $\bar{c}_t$. T-statistics for individual coefficients, the Wald test and the corresponding p-values are obtained using the reverse regression delta method. Row labeled "Rel.prob. (BIC)", where BIC is the Bayesian information criterion, gives the relative probability of a model i computed as $\exp\{(BIC_{\text{best}}-BIC_i)T/2\}$, where $BIC=\ln(\widehat{\sigma^2})+\ln(T)n/T$, n is the number of regressors, $\widehat{\sigma^2}=SSE/T$ of the regression, and T is the sample size. Relative probability of one indicates the best model selected by a given criterion. Relative probability of zero means that a given model has zero probability to explain the data equally well as the best model. Panel B reports the 5th and 95th percentiles of the $\bar{R}^2$ obtained under the null of EH from 10,000 Monte Carlo simulations of the model in Section 1. The parameters are $\delta_0=0, \delta_\tau=1.43, \delta_r=1$, and σ_τ, σ_r are calibrated to match st.dev.$(\tau_t)=1.90\%$ and st.dev.$(r_t)=1.74\%$ at each level of persistence of ϕ_τ, ϕ_r.

- Yields orthogonalized to trend inflation forecast bond returns
- R^2 higher than under the null of EH

Horce Race

- Single factor

$$\hat{cf}_t = \hat{\gamma}_0 + \hat{\gamma}_1 c_t^{(1)} + \hat{\gamma}_2 \bar{c}_t$$

Table 4
Predicting returns with the cycle factor

	$rx^{(2)}$	$rx^{(5)}$	$rx^{(7)}$	$rx^{(10)}$	$rx^{(15)}$	$rx^{(20)}$
A. Cycle factor						
$rx_{t+1}^{(n)} = \beta_0 + \beta_1 \widehat{cf}_t + \varepsilon_{t+1}^{(n)}$, where $\widehat{cf}_t = \hat{\gamma}_0 + \hat{\gamma}_1 c_t^{(1)} + \hat{\gamma}_2 \bar{c}_t$						
$\widehat{cf}_t$	0.62	0.68	0.70	0.73	0.74	0.72
	(3.80)	(4.64)	(4.92)	(5.16)	(5.26)	(5.06)
t-stat (SS,[5%,95%])	[1.31,4.27]	[2.05,4.91]	[2.38,5.16]	[2.56,5.38]	[2.72,5.47]	[2.66, 5.30]
$\bar{R}^2$	0.38	0.46	0.49	0.53	0.54	0.51
$\Delta\bar{R}^2$	0.02	0.01	0.01	0.01	0.01	0.04
B. Maturity-specific cycles						
B1. $rx_{t+1}^{(n)} = \alpha_0 + \alpha_1 c_t^{(1)} + \alpha_2 c_t^{(n)} + \varepsilon_{t+1}$						
$c_t^{(1)}$	−1.60	−0.93	−0.72	−0.54	−0.39	−0.31
	(−2.82)	(−3.67)	(−3.77)	(−3.71)	(−3.07)	(−2.55)
$c_t^{(n)}$	1.95	1.63	1.52	1.43	1.27	1.11
	(3.26)	(4.73)	(5.16)	(5.29)	(5.10)	(4.88)
$\bar{R}^2$	0.33	0.46	0.51	0.53	0.52	0.50
B2. $rx_{t+1}^{(n)} = \alpha_0 + \alpha_1 c_t^{(n)} + \varepsilon_{t+1}$						
$c_t^{(n)}$	0.13	0.36	0.49	0.63	0.76	0.76
	(2.24)	(2.94)	(3.34)	(3.57)	(3.77)	(3.88)
$\bar{R}^2$	0.04	0.11	0.16	0.23	0.30	0.34

Panel A shows the predictability of individual bond excess returns achieved with the cycle factor. The cycle factor $\widehat{cf}_t$ is defined in Equation (27). The row denoted "t-stat (SS,[5%,95%])" summarizes the small sample distributions of the reverse regression t-statistics obtained with nonparametric block bootstrap. Row $\Delta\bar{R}^2$ reports the difference in $\bar{R}^2$ between yields-plus-τ_t^{CPI} Regression (23) and the single-factor regression. Panel B presents regressions of individual excess returns on cycles of a given maturity. T-statistics in parentheses are obtained with the reverse regression delta method.

Yield Curve

$$y_t^{(n)} = \tilde{A}_n + \tilde{B}_n^{\tau} \tau_t^{CPI} + \tilde{B}_n^{r} c_t^{(1)} + \tilde{B}_n^{x} \hat{cf}_t + e_t^{(n)}$$

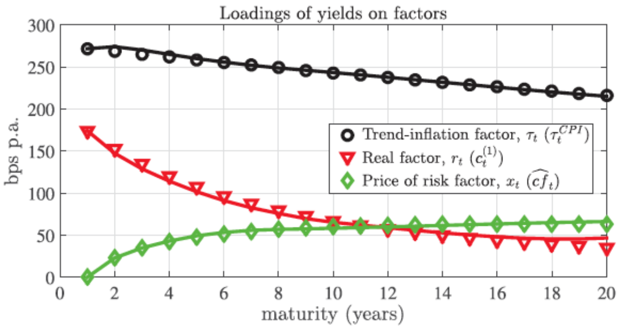


Figure 4
Loadings of yields on factors: regressions vs. affine model

The solid lines present the loadings of yields on observable factors $F_t = \left(\tau_t^{CPI}, c_t^{(1)}, \widehat{cf}_t\right)'$ obtained from Regression (30). The markers present the loadings obtained from the affine model given in Equation (17) for factors $F_t = (\tau_t, r_t, x_t)'$. The parameters of the affine model are calibrated by minimizing the sum of squared distances between the loadings from the regression and from the affine model, see Equations (31)–(32).

Comparing with Level, Slope and Curvature

- τ_t^{CPI} most correlated with the level factor
- Slope loads on both $c_t^{(1)}$ and $\hat{cf}_t$
  - $c_t^{(1)}$ does not forecast return, so **slope is a noisy measure of bond risk premium**

Horce Race: Cochrane Piazzesi (CP) Factor

Table 8
Bivariate predictive regressions with the CP and the cycle factor

A. CMT zero-coupon yields

	$rx^{(2)}$	$rx^{(5)}$	$rx^{(7)}$	$rx^{(10)}$	$rx^{(15)}$	$rx^{(20)}$
CP_t	−0.01	−0.05	−0.02	−0.03	0.00	0.07
	(−0.88)	(−0.41)	(−0.23)	(−0.18)	(0.03)	(0.36)
$\widehat{cf}_t$	0.63	0.71	0.72	0.75	0.74	0.67
	(3.55)	(3.84)	(3.93)	(4.05)	(3.99)	(3.63)
$\bar{R}^2$ (CP + $\widehat{cf}$)	0.38	0.46	0.49	0.53	0.54	0.51
R^2 from univariate regressions:						
R^2 (CP)	0.17	0.19	0.22	0.22	0.25	0.27
R^2 ($\widehat{cf}$)	0.38	0.46	0.49	0.53	0.54	0.51

B. Fama–Bliss zero-coupon yields

	$rx^{(2)}$	$rx^{(3)}$	$rx^{(4)}$	$rx^{(5)}$
CP_t	0.07	0.09	0.13	0.08
	(−0.28)	(0.09)	(0.37)	(0.22)
$\widehat{cf}_t$	0.56	0.56	0.56	0.59
	(4.02)	(4.21)	(4.36)	(4.51)
$\bar{R}^2$ (CP + $\widehat{cf}$)	0.36	0.39	0.41	0.41
R^2 from univariate regressions:				
R^2 (CP)	0.17	0.19	0.22	0.20
R^2 ($\widehat{cf}$)	0.36	0.38	0.41	0.41

Table 8 reports the results from bivariate predictive regressions of bond excess returns with the CP factor and the $\widehat{cf}$ factor as regressors: $rx_{t+1}^{(n)} = \alpha + \beta_1 CP_t + \beta_2 \widehat{cf}_t + \varepsilon_{t+1}$. The last two rows of each panel provide the predictive R^2 from univariate regressions using either the CP factor or the $\widehat{cf}$ factor as the regressor. Panel A uses CMT-based zero-coupon yields (forwards) with maturities of 1, 2, 5, 7, 10, and 20 years, to construct $\widehat{cf}_t$ and $CP_t = \hat{\gamma}' \mathbf{f}_t$ factors, as in Equations (27) and (35), respectively. Analogously, Panel B uses Fama-Bliss zero-coupon yields with maturities from 1 through 5 years to construct the forecasting factors. Reverse regression delta method t-statistics are in parentheses. All variables are standardized.

Out-of-Sample Forecast

Table 9
Out-of-sample tests

Test	$rx^{(2)}$	$rx^{(5)}$	$rx^{(7)}$	$rx^{(10)}$	$rx^{(15)}$	$rx^{(20)}$
A. Out-of-sample period: 1978–2011						
(1) ENC-NEW	139.69	146.19	157.23	176.97	176.40	157.61
(2) Bootstrap 95% CV	75.99	61.93	60.60	57.37	57.62	57.55
(3) MSE(6 cyc)/MSE(6 fwd)	0.70	0.63	0.60	0.56	0.56	0.62
(4) MSE(2 cyc)/MSE(6 fwd)	0.66	0.57	0.55	0.50	0.50	0.63
(5) R^2_{OOS}(6 cyc)	0.10	0.19	0.26	0.30	0.31	0.31
(6) R^2_{OOS}(2 cyc)	0.16	0.27	0.33	0.38	0.38	0.30
(7) R^2_{OOS}(6 fwd)	−0.29	−0.28	−0.23	−0.25	−0.24	−0.12
(8) R^2_{OOS}(fwd-spot spread)	0.10	0.09	0.07	0.08	0.09	0.09
B. Out-of-sample period: 1985–2011						
(1) ENC-NEW	116.49	115.58	127.23	145.34	146.33	134.47
(2) Bootstrap 95% CV	48.52	39.18	39.04	39.81	41.76	44.06
(3) MSE(6 cyc)/MSE(6 fwd)	0.69	0.64	0.62	0.60	0.62	0.67
(4) MSE(2 cyc)/MSE(6 fwd)	0.59	0.52	0.52	0.50	0.52	0.59
(5) R^2_{OOS}(6 cyc)	−0.08	0.16	0.25	0.30	0.31	0.31
(6) R^2_{OOS}(2 cyc)	0.08	0.31	0.37	0.42	0.42	0.39
(7) R^2_{OOS}(6 fwd)	−0.55	−0.31	−0.20	−0.17	−0.11	−0.04
(8) R^2_{OOS}(fwd-spot spread)	0.07	0.07	0.05	0.08	0.09	0.10
C. Out-of-sample period: 1995–2011						
(1) ENC-NEW	61.65	57.88	66.75	81.24	86.43	81.53
(2) Bootstrap 95% CV	22.28	23.11	25.34	24.53	27.28	36.22
(3) MSE(6 cyc)/MSE(6 fwd)	0.67	0.64	0.59	0.52	0.50	0.54
(4) MSE(2 cyc)/MSE(6 fwd)	0.46	0.46	0.42	0.41	0.38	0.33
(5) R^2_{OOS}(6 cyc)	−0.58	−0.26	−0.16	−0.03	0.01	−0.12
(6) R^2_{OOS}(2 cyc)	−0.09	0.10	0.17	0.20	0.26	0.32
(7) R^2_{OOS}(6 fwd)	−1.37	−0.97	−0.97	−0.97	−0.97	−1.07
(8) R^2_{OOS}(fwd-spot spread)	−0.19	−0.06	−0.09	−0.06	−0.06	−0.06

Table 9 reports the results of out-of-sample tests. Row (1) in each panel contains the ENC-NEW test. The null hypothesis is that the predictive regression with forward rates encompasses all predictability in bond excess returns. The null is tested against the alternative that cycles improve the predictability achieved by forward rates. For forward rates and cycles we use maturities of 1, 2, 5, 7, 10, and 20 years. Row (2) reports bootstrapped critical values (CV) for the ENC-NEW statistic at the 95% confidence level (see the Online Appendix for implementation details). Rows (3) and (4) show the ratio of mean squared errors for the cycles and forward-rate models, Row (3) with six cycles and Row (4) with two cycles, $\bar{c}_t, c_t^{(1)}$. Rows (5)–(8) report the out-of-sample R^2, R^2_{OOS}, for two and six cycles, forwards rates and the forward-spot spread, respectively. For forward-rate and six-cycles specifications we use the same six maturities, for two cycles we use $c_t^{(1)}$ and $\bar{c}_t$ as in the construction of forecasting factor $\widehat{cf}_t$. The forward-spot spread used for predicting the bond return with maturity n is constructed as $f_t^{(n)} - y_t^{(1)}$.

Cyclicality of Bond Risk Premium: Ludvigson and Ng (2009, RFS)

- Do bond risk premiums move with macroeconomic factors? If so, how?
- Dynamic factor model to extract macro factors from more than 100 macro series

Predictive Regression

Table 2
Regression of monthly excess bond returns on lagged cactors

Model: $rx_{t+1}^{(n)} = \beta_0 + \beta_1' \widehat{F}_t + \beta_2 CP_t + \epsilon_{t+1},$

		$\widehat{F}_{1t}$	$\widehat{F}_{1t}^3$	$\widehat{F}_{2t}$	$\widehat{F}_{3t}$	$\widehat{F}_{4t}$	$\widehat{F}_{8t}$	CP_t	$F5_t$	$F6_t$	R^2
	(a)							**0.45**			0.31
								(8.90)			
	(b)	**−0.93**	**0.06**	**−0.40**	**0.18**	**−0.33**	**0.35**				0.26
		(−5.19)	(2.78)	(−3.10)	(2.24)	(−2.94)	(4.35)				
	(c)	**−0.74**	**0.05**	0.08	**0.24**	**−0.24**	**0.24**	**0.41**			0.45
		(−4.48)	(2.70)	(0.71)	(3.84)	(−2.51)	(2.70)	(5.22)			
$rx_{t+1}^{(2)}$	(d)	**−0.93**	**0.06**		0.18	**−0.33**	**0.35**				0.22
		(−4.96)	(2.87)		(1.87)	(−2.65)	(3.83)				
	(e)	**−0.75**	**0.05**		**0.24**	**−0.25**	**0.24**	**0.40**			0.45
		(−4.71)	(2.71)		(3.85)	(−2.61)	(2.89)	(5.89)			
	(f)								**0.54**		0.22
									(5.52)		
	(g)									**0.50**	0.26
										(6.78)	
	(h)							**0.39**	**0.43**		0.44
								(6.0)	(5.78)		
	(a)							**0.85**			0.34
								(8.52)			
	(b)	**−1.59**	**0.11**		0.19	**−0.53**	**0.64**				0.18
		(−4.68)	(3.12)		(1.05)	(−2.23)	(3.73)				
	(c)	**−1.22**	**0.10**		**0.30**	**−0.36**	**0.44**	**0.76**			0.44
		(−4.39)	(2.96)		(2.78)	(−2.12)	(2.74)	(6.13)			
$rx_{t+1}^{(3)}$	(d)								**0.91**		0.19
									(5.28)		
	(e)									**0.89**	0.24
										(6.57)	
	(f)							**0.75**	**0.69**		0.44
								(6.16)	(5.55)		
	(a)							**1.24**			0.37
								(8.58)			
	(b)	**−2.05**	**0.16**		0.18	−0.63	**0.95**				0.16
		(−4.49)	(3.20)		(0.68)	(−1.77)	(3.75)				
	(c)	**−1.51**	**0.14**		**0.35**	−0.37	**0.64**	**1.13**			0.45
		(−4.20)	(3.08)		(2.22)	(−1.50)	(2.83)	(6.40)			
$rx_{t+1}^{(4)}$	(d)								**1.19**		0.17
									(5.08)		
	(e)									**1.20**	0.23
										(6.57)	
	(f)							**1.11**	**0.87**		0.45
								(6.30)	(5.39)		
	(a)							**1.46**			0.34
								(7.90)			
	(b)	**−2.27**	**0.18**		0.18	−0.78	**1.13**				0.14
		(−4.10)	(3.06)		(0.55)	(−1.80)	(3.68)				
	(c)	**−1.63**	**0.15**		**0.38**	−0.48	**0.76**	**1.34**			0.41
		(−3.86)	(2.95)		(1.92)	(−1.54)	(2.76)	(6.00)			
$rx_{t+1}^{(5)}$	(d)								**1.36**		0.14
									(4.80)		
	(e)									**1.41**	0.21
										(6.47)	
	(f)							**1.32**	**0.98**		0.42
								(5.87)	(5.08)		

Notes: The table reports estimates from OLS regressions of excess bond returns on the lagged variables named in column 1. The dependent variable $rx_{t+1}^{(n)}$ is the excess log return on the n-year Treasury bond. $\widehat{F}_t$ denotes a set of regressors including $F5_t$, $F6_t$, and $\widehat{F}_{it}$. These denote factors estimated by the method of principal components using a panel of data with 132 individual series over the period 1964:1–2003:12. $F5_t$ is the single factor constructed as a linear combination of the five estimated factors $\widehat{F}_{1t}$, $\widehat{F}_{1t}^3$, $\widehat{F}_{3t}$, $\widehat{F}_{4t}$, and $\widehat{F}_{8t}$. $F6_t$, is the single factor constructed as a linear combination of the six estimated factors $\widehat{F}_{1t}$, $\widehat{F}_{2t}$, $\widehat{F}_{1t}^3$, $\widehat{F}_{3t}$ $\widehat{F}_{4t}$, and $\widehat{F}_{8t}$. CP_t is the Cochrane and Piazzesi (2005) factor that is a linear combination of five forward spreads. Newey and West (1987) corrected t-statistics have lag order 18 months and are reported in parentheses. Coefficients that are statistically significant at the 5% or better level are highlighted in bold. A constant is always included in the regression even though its estimate is not reported in the table. The sample spans the period 1964:1–2003:12.

Countercyclical Bond Risk Premium

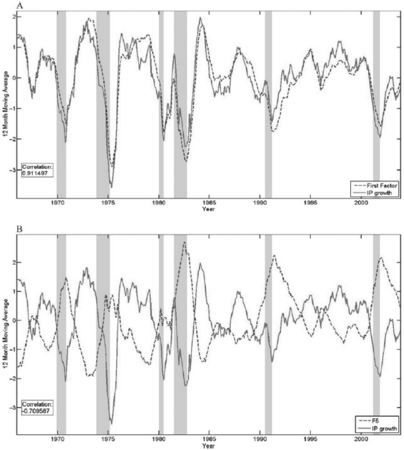


Figure 6
A: First factor and IP growth. B: *F5* and IP growth
Note: Standardized units are reported. Shadings denote months designated as recessions by the National Bureau of Economic Research. "First factor" denotes the first estimated factor, F_1t. $F5$ denotes the linear combination of five factors, written in the text as $F5_t$.

The Importance of Including Macro Risk Factors

- Excluding macro risk factors, bond risk premium is acyclical

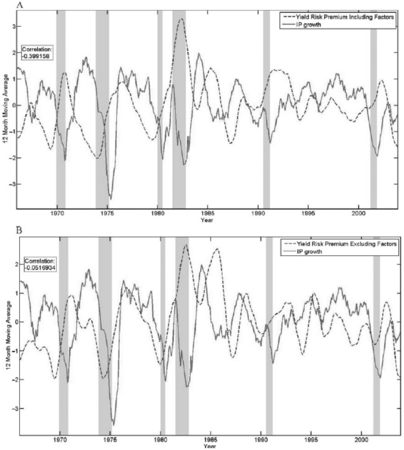


Figure 7
A: Yield risk premium including factors and IP growth. B: Yield risk premium excluding factors and IP growth (standardized).
Note: Standardized units are reported. Shadings denote months designated as recessions by the National Bureau of Economic Research.

The Spanning Puzzle: Joslin, Prebsch and Singleton (2014, JF)

- In a general framework in which common latent factors drive macro variables and bond yields, we need the same number of yields as latent factors to back out the latent factors
- Put differently, a combination of yields should span the latent factors and thus the predictable component of macroeconomic variables
- Testable implications
  - Projecting macro variables onto yields should lead to serially uncorrelated residuals
  - Yields should have high explanatory power on the predictable component of macro variables (through predictive regression)

Residual Serial Correlation

Table 2 Projections of economic activity and inflation on nominal yields. Five measures capture the output gap. Five others capture current economic growth. They are all described in the notes to Table 1. Three quarterly inflation measures are the quarter-to-quarter log change in the GDP deflator, the log change in the CPI (final month in previous quarter to final month in current quarter), and the log change in the CPI excluding food and energy. This table reports R^2s and first-order serial correlation of residuals for cross-sectional regressions on six Treasury bond yields. The bond maturities range from three months to five years. There are missing observations for parts of the sample periods for capacity utilization, CFNAI, and the Ludvigson-Ng principal component. The available data ranges for the economic activity data are described in the notes to Table 1. Inflation measured with the CPI excluding food and energy begins with 1957Q2

	1952Q2–2010Q4		1986Q1–2007Q4	
Measure	R^2	AR(1) of residuals	R^2	AR(1) of residuals
Output gap				
log real GDP, detrended	0.38	0.93	0.41	0.83
log ind prod, detrended	0.44	0.91	0.33	0.89
log real GDP, HP-filtered	0.33	0.73	0.54	0.67
log ind prod, HP-filtered	0.34	0.70	0.62	0.68
demeaned capacity use	0.42	0.83	0.64	0.79
Growth				
log-diff per capita C	0.09	0.30	0.07	0.01
log-diff real GDP	0.02	0.34	0.19	0.15
log-diff ind prod	0.03	0.35	0.26	0.11
CFNAI	0.44	0.71	0.33	0.48
Ludvigson-Ng PC	0.33	0.60	0.54	0.31
Inflation				
GDP deflator	0.50	0.82	0.32	0.72
CPI	0.62	0.77	0.43	0.56
CPI ex food & energy	0.62	0.81	0.82	0.66

R^2: Yields and Predictable Components of Macro Variables

Table 3 Projections of expected future economic growth on the nominal term structure. Measures of economic growth in quarter $t+2$ are regressed on economic growth in quarters t and $t-1$, CPI inflation in quarters t and $t-1$, and six quarter-t Treasury bond yields. The fitted values of these forecasting regressions are regressed on the six Treasury quarter-t bond yields. The R^2 of the second regression is the fraction of the regression-based expectations that is spanned by the nominal term structure. The table reports the R^2s of both regressions. It also reports the serial correlation of the residuals from the second-stage regression. The five measures of economic growth and their available data ranges are described in the notes to Table 1

	Forecasting regression R^2	Fraction of forecast spanned by nominal yields	AR(1) of unspanned component
1952Q2–2010Q4			
log-diff per capita C	0.24	0.68	0.44
log-diff real GDP	0.15	0.71	0.15
log-diff ind prod	0.15	0.80	0.40
CFNAI	0.39	0.34	0.73
Ludvigson-Ng PC	0.41	0.17	0.61
1986Q1–2007Q4			
log-diff per capita C	0.20	0.30	0.19
log-diff real GDP	0.17	0.43	0.04
log-diff ind prod	0.19	0.53	0.14
CFNAI	0.44	0.35	0.55
Ludvigson-Ng PC	0.36	0.68	0.43

- Yields do not span the predictable component of macro variables

Knife-edge Restriction

- A possible explanation: macro variables affect both expected short rate and bond risk premium, whose effects offset
- On this explanation: Bauer and Rudebusch (2018, RF)

Do Macro Variables Span Yields?

Table 4 Projections of nominal yields on measures of economic activity and inflation. Quarter-end yields on Treasury bonds with maturities ranging from three months to five years are regressed on two contemporaneous measures of the output gap, lags zero through two of ARMA-smoothed CPI inflation, and lags zero through two of the log-change in industrial production. This table reports R^2s, standard deviations of the fitted residuals, and first-order serial correlation of residuals

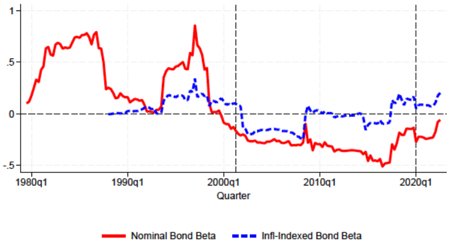
	1952Q2–2010Q4			1986Q1–2007Q4		
Maturity	R^2	Std dev of residuals	AR(1) of residuals	R^2	Std dev of residuals	AR(1) of residuals
Three months	0.66	1.74	0.83	0.56	1.27	0.86
One year	0.64	1.79	0.86	0.58	1.29	0.83
Two years	0.62	1.83	0.87	0.55	1.28	0.81
Three years	0.59	1.83	0.88	0.52	1.26	0.81
Four years	0.58	1.82	0.89	0.52	1.23	0.81
Five years	0.58	1.80	0.90	0.51	1.19	0.82

- Macro variables account for about 60-70% of yield variance
- May be related to future macro variables, need more economic structure

Macro Risks in Stocks and Bonds: Stock-bond Correlation

- Stock-bond correlation was positive before 2000, and turned negative afterwards until Covid, and reverted recently

Figure 1: Rolling Nominal and Real Bond-Stock Betas



Note: This figure shows betas from regressing quarterly ten-year Treasury bond excess returns onto quarterly US equity excess returns over five-year rolling windows for the period 1979.Q4-2022.Q3. Quarterly excess returns are in excess of three-month T-bills. Prior to 1999, I replace US Treasury Inflation Protected (TIPS) returns with UK ten-year linker returns. Bond excess returns are computed from changes in yields. Zero-coupon yield curves from Gürkaynak, Sack and Wright (2006, 2008) and the Bank of England. Vertical lines indicate 2001.Q2 and the start of the pandemic 2020.Q1.

Source: Pflueger (2024)

Why Should We Care?

- A big deal for asset allocation: How to invest in stocks and bonds jointly?
- Identify changing underlying macroeconomic environment/risks
- ...

Related Literature

- Campbell, Sunderam and Viceira (2017, CFR) and Campebll, Pflueger and Viceira (2020, JPE): time-varying real-nominal covariance
- Song (2017, RFS): changing MP regime and changing inflation cyclicality
- Chernov, Lochstoer and Song (2023): changing transitory vs permanent component of productivity
- Fang, Liu and Roussanov (2024): changing cyclicality of energy inflation
- Pflueger (2024): MP regime + supply/demand dominance shift

3. Structural models of bond prices

Roadmap

- A New-Keynesian term structure model
- Structural model of SDF (consumption-based models)
- General equilibrium with both consumption and production

NK Model: Bekaert, Cho and Moreno (2010, JMCB)

- Term structure is the key to understanding monetary policy transmission
  - Monetary policy moves the short rate, but the cost of capital is a long rate
  - Most macro models abstract it away by assuming expectation hypothesis
- This paper: connect the NK model and no-arbitrage term structure models
  - Using bond yields to identify macro parameters and latent factors (inflation target and natural rate of output)
  - A restricted affine model that follows NK decision rule

The Full Model

- Philips curve

$$\pi_t = \delta E_t \pi_{t+1} + (1-\delta)\pi_{t-1} + \kappa(y_t - y_t^n) + \varepsilon_{AS,t}$$

- IS curve

$$y_t = \alpha_{IS} + \mu E_t y_{t+1} + (1-\mu) y_{t-1} - \phi(i_t - E_t \pi_{t+1}) + \varepsilon_{IS,t}$$

- Generalized Taylor rule

$$i_t = \alpha_{MP} + \rho i_{t-1} + (1-\rho)[\beta(E_t \pi_{t+1} - \pi_t^*) + \gamma(y_t - y_t^n)] + \varepsilon_{MP,t}$$

- Dynamics of latent factors

$$y_t^n = \lambda y_{t-1}^n + \varepsilon_{y^n,t}$$

$$\pi_t^* = \varphi_1 E_t \pi_{t+1}^* + \varphi_2 \pi_{t-1}^* + \varphi_3 \pi_t + \varepsilon_{\pi^*,t}$$

The Full Term Structure Model

- State dynamics from the NK model

$$x_t = c + \Omega x_{t-1} + \Gamma \varepsilon_t$$

where the VAR coefficients are restricted by the NK structural parameters

- SDF

$$m_{t+1} = \ln \psi - \sigma y_{t+1} + (\sigma + \eta) y_t - \eta y_{t-1} + (g_{t+1} - g_t) - \pi_{t+1}$$

where g_t is an aggregate demand shock

- Bond yields are affine function of x_t and the model can be estimated using more information from bond yields

Consumption-based Models

- A fundamental result: Backus, Gregory and Zin (1989)
- Habit formation: Wachter (2006)
- LRR: Bansal and Shaliastovich (2013)

Backus, Gregory and Zin (1989) Result

- Matching the observed positive bond risk premium would require aggregate consumption growth to have a counterfactually large and negative autocorrelation
- Intuition: risk-free rate increases when expected consumption growth is high. To have a positive bond risk premium, expected consumption growth must be low (bond price is high) when SDF is low (consumption growth is high)
- Require a negative correlation of consumption growth

Habit: Wachter (2006, JFE)

- Preference
$$E\sum_{t=0}^{\infty}\frac{(C_t-X_t)^{1-\gamma}-1}{1-\gamma}$$
where X_t is defined indirectly through surplus consumption
$$S_t \equiv \frac{C_t-X_t}{C_t}$$
- Habit specification
$$s_{t+1}=(1-\rho)\bar{s}+\phi s_t+\lambda(s_t)(\Delta c_{t+1}-E_t\Delta c_{t+1})$$
- Consumption dynamics
$$\Delta c_{t+1}=g+v_{t+1}$$
- Risk-free rate
$$r_{f,t}=-\ln\delta+\gamma g+\gamma(1-\phi)(\bar{s}-s_t)-\frac{\gamma^2\sigma_v^2}{2}(1+\lambda(s_t))^2$$
$\lambda(s_t)$ controls the SDF volatility

Specifying $\lambda(s_t)$

- Parameterize $\lambda(s_t)$ as

$$\lambda(s_t) = (1/\bar{S})\sqrt{1 - 2(s_t - \bar{s})} - 1$$

where

$$\bar{S} = \sigma_v \sqrt{\frac{\gamma}{1 - \phi - b/\gamma}}$$

Thus,

$$r_{f,t} = -\ln\delta + \gamma g - \frac{\gamma(1-\phi) - b}{2} + b(\bar{s} - s_t)$$

- The role of b
  - Campbell and Cochrane (1999): $b = 0$, constant interest rate
  - $b > 0$: low interest rate when s_t is high, making bonds risky
    - Intertemporal smoothing > precautionary saving effect

Consumption and Inflation Dynamics

$$\Delta c_{t+1} = (1 - \psi_1)g + \psi_1 \Delta c_t + \theta_1 v_{1,t} + v_{1,t+1}$$

$$\Delta \pi_{t+1} = (1 - \psi_2)\bar{\pi} + \psi_2 \Delta \pi_t + \theta_2 v_{2,t} + v_{2,t+1}$$

where $v_{1,t}, v_{2,t}$ have correlation ρ. Estimating the model leads to $\rho = -0.205$.

Results: Average Yield Curve

Table 4
Means and standard deviations of continuously compounded zero-coupon bond yields in the model and in the data

Columns marked "Real" give statistics for real yields on real bonds (bonds that pay off in units of aggregate consumption) in the model; columns market "Nominal" give statistics for nominal yields on nominal bonds in the model; columns marked "Data" give statistics for nominal yields on nominal bonds in the data. Yields are in annual percentages. Maturity is in quarters. Data are quarterly, begin in the second quarter of 1952, and end in the third quarter of 2004.

Maturity	Mean			Stand. dev.		
	Real	Nominal	Data	Real	Nominal	Data
1	1.46	5.17	5.22	1.91	2.35	2.93
4	1.62	5.34	5.60	1.96	2.35	2.93
8	1.83	5.58	5.81	2.03	2.37	2.89
12	2.05	5.83	5.98	2.10	2.40	2.82
16	2.28	6.07	6.11	2.17	2.44	2.79
20	2.51	6.32	6.19	2.24	2.48	2.74

- Both the nominal and real yield curves slope up

Results: Return Predictability

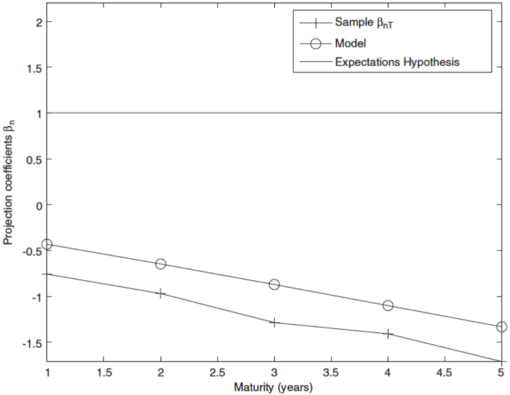


Fig. 6. Long-rate regressions. Coefficients β_n from the regression

$$y^{\$}_{n-1,t+1} - y^{\$}_{tn} = \alpha_n + \beta_n \frac{1}{n-1}(y^{\$}_{nt} - y^{\$}_{1t}) + \text{error}$$

using simulated and actual data on bond yields. Lines with circles denote coefficients implied by simulated data; lines with plus signs denote coefficients implied by actual data. Parameter values are as in Tables 1 and 2. The solid line denotes the coefficients if the expectations hypothesis were to hold. Data are quarterly, begin in the second quarter of 1952, and end in the third quarter of 2004.

Inflation and the Real Economy: Piazzesi and Schneider (2006, NBER Macro Annual)

- One way to obtain an upward-sloped yield curve in a standard model (without habit) is to assume a negative correlation between expected inflation and expected consumption growth
- Nominal bond prices in bad times
  - Higher because expected growth rate is low
  - Lower because expected inflation is high
- The second dominates the first, making nominal bonds risky

LRR: Bansal and Shaliastovich (2013, RFS)

- Representative agents with Epstein-Zin utility: real SDF

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1}$$

- Macroeconomic dynamics

$$\Delta c_{t+1} = \mu_c + x_{ct} + \sigma_c \eta_{c,t+1}$$

$$\pi_{t+1} = \mu_\pi + x_{\pi t} + \sigma_\pi \eta_{\pi,t+1}$$

- x_t follows

$$x_{t+1} = \Pi x_t + \Sigma_t e_{t+1}$$

  where

$$\Pi = \begin{bmatrix} \rho_c & \rho_{c\pi} \\ 0 & \rho_\pi \end{bmatrix}, \Sigma_t = \begin{bmatrix} \sigma_{ct} & 0 \\ 0 & \sigma_{\pi t} \end{bmatrix}$$

  Importantly, $\rho_{c\pi} < 0$, i.e., higher expected inflation lowers future expected growth

- σ_{ct} and $\sigma_{\pi t}$ follow AR(1) processes

Bond Markets: Data and Model

Table 6
Bond markets: U.S. data and model

	1y	2y	3y	4y	5y
Data:					
Yield Level	6.09	6.33	6.52	6.68	6.79
Std. Dev.	3.09	2.97	2.87	2.78	2.70
EH Slope:		−0.41	−0.78	−1.14	−1.15
		(0.44)	(0.56)	(0.63)	(0.67)
CP Slope:		0.44	0.85	1.28	1.43
		(0.11)	(0.23)	(0.33)	(0.42)
CP R^2		0.15	0.17	0.20	0.17
		(0.10)	(0.10)	(0.11)	(0.12)
Model:					
Yield Level:	6.10	6.29	6.50	6.73	6.97
Std. Dev.	2.37	2.29	2.24	2.20	2.17
EH Slope:		−0.45	−0.51	−0.57	−0.61
CP Slope		0.44	0.83	1.19	1.53
CP R^2		0.16	0.17	0.17	0.16

Nominal term structure, slopes in the expectations hypothesis regressions, and slopes and R^2s in Cochrane and Piazzesi (2005) single-factor bond premium regressions. Data are second-month-of-the-quarter observations of quarterly yields from 1969 to 2010; model output is based on population values.

Endogenous Growth Model: Kung (2015, JFE)

- Why are expected inflation and expected growth negative correlated?
- Through endogenous growth
  - Negative TFP shock lowers current output and raises current and expected inflation
  - Because the expected future profit of R&D is lower, firms spend less in R&D and thus harm long-run growth
- Bansal-Shaliastovich + NK + Romer

Model Setup

- Households

$$U_t = \left\{ (1-\beta)(C_t^*)^{\frac{1-\gamma}{\theta}} + \beta \left[E_t(U_{t+1}^{1-\gamma}) \right]^{\frac{1}{\theta}} \right\}^{\theta/(1-\gamma)}$$

  where $C_t^* = C_t(\bar{L} - L_t)^\tau$

- Final goods producers

$$Y_t = \left(\int_0^1 X_{i,t}^{(\nu-1)/\nu} di \right)^{\nu/(\nu-1)}$$

- Intermediate good firms

$$X_{i,t} = K_{i,t}^\alpha (Z_{i,t} L_{i,t})^{1-\alpha}$$

  where $N_{i,t}$ has externalities

$$Z_{i,t} = A_t N_{i,t}^\eta N_t^{1-\eta}$$

- $a_t \equiv \log A_t$ and σ_t^2 follow exogenous AR(1) processes

Model Setup (Cont'ed)

- Intermediate good firms
  - Physical capital dynamics

$$K_{i,t+1} = (1-\delta_k)K_{i,t} + \Phi_k(\frac{I_{i,t}}{K_{i,t}})K_{i,t}$$

where $\Phi_k(\frac{I_{i,t}}{K_{i,t}}) = \frac{\alpha_{1,k}}{1-1/\zeta_k}\left(\frac{I_{i,t}}{K_{i,t}}\right)^{1-1/\zeta_k} + \alpha_{2,k}$

  - R&D stock dynamics

$$N_{i,t+1} = (1-\delta_n)N_{i,t} + \Phi_n(\frac{S_{i,t}}{N_{i,t}})N_{i,t}$$

where $\Phi_n(\frac{S_{i,t}}{N_{i,t}}) = \frac{\alpha_{1,n}}{1-1/\zeta_n}\left(\frac{S_{i,t}}{N_{i,t}}\right)^{1-1/\zeta_n} + \alpha_{2,n}$

  - Rotemberg pricing

$$G(P_{i,t}, P_{i,t-1}; P_t, Y_t) = \frac{\phi_R}{2}\left(\frac{P_{i,t}}{\Pi_{ss}P_{i,t-1}} - 1\right)^2 Y_t$$

  - Firms' choice variables: physical capital investment $I_{i,t}$, R&D investment $S_{i,t}$, labor demand $L_{i,t}$ and price setting $P_{i,t}$

Model Setup (Cont'ed)

- Central bank

$$\ln\left(\frac{R_{t+1}}{R_{ss}}\right) = \rho_r \ln\left(\frac{R_t}{R_{ss}}\right) + (1-\rho_r)\left(\rho_\pi \ln\left(\frac{\Pi_t}{\Pi_{ss}}\right) + \rho_y \ln\left(\frac{\hat{Y}_t}{Y_{ss}}\right)\right) + \sigma_\xi \xi_t$$

- Equilibrium
  - All intermediate firms are identical, i.e., $P_{i,t} = P_t$, $X_{i,t} = X_t$, $K_{i,t} = K_t$, $L_{i,t} = L_t$, $N_{i,t} = N_t$, $I_{i,t} = I_t$, $S_{i,t} = S_t$
  - Resource constraint

$$Y_t = C_t + S_t + I + t + \frac{\phi_R}{2}\left(\frac{\Pi_t}{\Pi_{ss}} - 1\right)^2 Y_t$$

Asset Pricing Moments

Table 6
Asset pricing moments.

This table reports the means, standard deviations, and correlations for key asset pricing variables, such as the return on the equity claim r_d, the real risk-free rate r_f, the five-year minus one-quarter yield spread $y^{(5)} - y^{(1Q)}$, and the nominal short rate $y^{(1Q)}$, for the data and the model. The model is calibrated at a quarterly frequency, and the reported statistics are annualized. Low-frequency components are obtained using a bandpass filter and isolating frequencies between 20 and 50 years.

Statistic	Data	Model
Panel A : Means		
$E(r_d - r_f)$ (percent)	5.84	3.17
$E(r_f)$ (percent)	1.62	1.07
$E(y^{(5)} - y^{(1Q)})$ (percent)	1.02	0.96
$E(y^{(1Q)})$ (percent)	5.03	5.05
Panel B : Standard deviations		
$\sigma(r_d - r_f)$ (percent)	17.87	6.68
$\sigma(r_f)$ (percent)	0.67	0.68
$\sigma(y^{(5)} - y^{(1Q)})$ (percent)	1.05	1.08
$\sigma(y^{(1Q)})$ (percent)	2.96	3.09
Panel C : Correlations		
$corr(y^{(5)} - y^{(1Q)}, \pi)$	−0.40	−0.46
$corr(y^{(5)} - y^{(1Q)}, \pi)$ (low-frequency)	−0.69	−0.54
$corr(y^{(5)} - y^{(1Q)}, s - n)$ (low-frequency)	0.72	0.77

Other Production-based Models

- Rudebusch and Swanson (2008 JME, 2012 AEJ Macro)
- Jermann (2013 JFE)
- van Binsbergen et al (2012 JME)

The Role of Inflation: Duffee (2018, JF)

- Building block of generating upward-sloped yield curve: negative correlation between expected inflation and growth
- Duffee (2018) questions the quantitative importance of expected inflation on explaining bond yields
  - The variance of shocks to expected inflation to the variance of yields is much smaller than most macro-finance models predict
  - Nominal yield news: mostly news on real rate or risk premia

4. Preferred-habitat models

Vayanos and Vila (2021, ECMA)

Two Views of the Term Structure of Interest Rates

- Consumption-based view: intertemporal rate of substitution
- Preferred-habitat view: investor clienteles for specific maturity
  - A degree of segmentation
- Examples interpreted with the preferred-habitat view
  - 2004 UK pension reform: Pension funds bought long-maturity bonds in large quantities and supresses long-maturity yields
  - QE by major central banks: large-scale purchases of long-maturity bonds
- Shortcoming: extreme form of preferred-habitat view leaves large arbitrage profits

This Paper: Research Questions

- How do shocks to clientele demands affect term structure?
- What's the effect of large-scale central bank bond purchases?
- What are implications of the preferred-habitat view for interest rate dynamics, bond risk premia, and monetary transmission?

What This Paper Does

- A model that formalizes the preferred-habitat view and embeds into a modern no-arbitrage framework
- Answer research questions both qualitatively and quantitatively
  - Replicate basic patterns of interest rates: average slope, return predictability
  - How does monetary policy transmiss to the long rate?
  - How does the demand of bonds at certain maturities affect the whole yield curve? More specifically, what is the effect of QE?

Relation to the Literature

- Formalize the preferred-habitat theory of term structure
- Portfolio balance channel in macroeconomic models
- Demand shocks drive asset prices
- A special affine no-arbitrage term structure model

Model Setup: Preferred Habitat Investors

- For each maturity τ, the demand of preferred habitat investors

$$Z_t^{(\tau)} = -\alpha(\tau) \log P_t^{(\tau)} - \beta_t^{(\tau)}$$

  - $Z_t^{(\tau)}$ is in present value
  - $\alpha(\tau)$ is the price elasticity
  - $\beta_t^{(\tau)}$ is the intercept of the demand function. A higher β_t^{τ} means a lower demand for preferred habitat investors

$$\beta_t^{(\tau)} = \theta_0(\tau) + \sum_{k=1}^{K} \theta_k(\tau)\beta_{k,t}$$

Model Setup: Arbitrageurs

$$\max_{X_t^\tau} \left[E_t(dW_t) - \frac{a}{2} var_t(dW_t) \right]$$

$$s.t. : dW_t = \left(W_t - \int_0^\infty X_t^{(\tau)} d\tau \right) r_t dt + \int_0^\infty X_t^{(\tau)} \frac{dP_t^{(\tau)}}{P_t^{(\tau)}}$$

where $P_t^{(\tau)}$ is the price of bond with maturity τ. Notice that P is not only a function of state variables but also τ and

$$dP_t^{(\tau)} = P_{t+dt}^{(\tau-dt)} - P_t^{(\tau)}$$

Short Rate Model

- Mute demand risk, $\theta_k(\tau) = 0$ for all k and $\beta_t^{(\tau)} = \theta_0^{(\tau)}$
- Short rate is mean-reverting

$$dr_t = \kappa_r(\bar{r} - r_t)dt + \sigma_r dB_{r,t}$$

- Equilibrium conjecture (affine solution!)

$$P_t^{(\tau)} = \exp\left(-A_r(\tau)r_t + C(\tau)\right)$$

- Bond return dynamics

$$\frac{dP_t^{(\tau)}}{P_t^{(\tau)}} = \mu_t^{(\tau)} dt - A_r(\tau)\sigma_r dB_{r,t}$$

where

$$\mu_t^{(r)} = A_r^{'}(\tau)r_t + C^{'}(\tau) + A_r(\tau)\kappa_r(r_t - \bar{r}) + \frac{1}{2}A_r(\tau)^2\sigma_r^2$$

Bond Risk Premia

$$\mu_t^{(\tau)} - r_t = -A_r(\tau)\lambda_{r,t}$$

where

$$\lambda_{r,t} = -a\sigma_r^2 \int_0^\infty X_t^{(\tau)} A_r(\tau) d\tau$$

- a: risk aversion
- $\int_0^\infty X_t^{(\tau)} A_r(\tau) d\tau$: the transmission of short rate shock to SDF (portfolio return) shock

Market Clearing and Solution in ODEs

$$X_t^{(\tau)} + Z_t^{(\tau)} = 0$$

Substitute the market clearing condition into $\lambda_{r,t}$:

$$-A_r(\tau)\lambda_{r,t} = A_r^{'}(\tau)r_t + C^{'}(\tau) + A_r(\tau)\kappa_r(r_t - \bar{r}) + \frac{1}{2}A_r(\tau)^2\sigma_r^2 - r_t$$

where

$$\lambda_{r,t} = -a\sigma_r^2 \int_0^\infty \alpha(\tau)\left([A_r(\tau)r_t + C(\tau)] - \theta_0(\tau)\right)A_r(\tau)d\tau$$

The constant and coefficient on r should be identifical, so that

$$A_r^{'}(\tau) + \kappa_r A_r(\tau) - 1 = -a\sigma_r^2 A_r(\tau)\int_0^\infty \alpha(\tau)A_r(\tau)^2 d\tau$$

$$C^{'}(\tau) - \kappa_r\bar{r}A_r(\tau) + \frac{1}{2}\sigma_r^2 A_r(\tau)^2 = a\sigma_r^2 A_r(\tau)\int_0^\infty [\theta_0(\tau) - \alpha(\tau)C(\tau)]\,A_r(\tau)d\tau$$

Boundary conditions $A_r(0) = C(0) = 0$

The Transmission of Monetary Policy

- When short rate drops, arbitrageurs are incentivized to do carry trade and increase their holding of long bonds, which lowers long rate
- Long rate response smaller than short rate as arbitraguers require compensation for bearing interest rate risks
- Match the relation between yield curve slope and risk premia

Global Demand Effect

What happens when the demand intercept $\theta_0(\tau)$ changes to $\theta_0(\tau) + \Delta\theta_0(\tau)$?

- The only place where $\theta_0(\tau)$ enters the solution is in the price of risk
- Price of risk does not depend on $\theta_0(\tau)$ only, but on $\int_0^\infty A_r(\tau)\theta_0(\tau)d\tau$
  - The origin of maturity-specific demand change does not matter as long as its effect on the price of risk is the same
  - Global effect on **all** yields

Demand Risk Model

- Introduce demand risks, assuming mean-reverting K demand factors (zero-mean) and short rate with

$$dq_t = -\Gamma(q_r - \bar{r}\mathcal{E})dt + \Sigma dB_t$$

- Same solution procedure, more complicated math, skipped

The Transmission of Monetary Policy

- Demand risk weakens the transmission of monetary policy
  - Carry trade is riskier
- As long-maturity bonds are more exposed to demand shocks, investors may even hold negative long-term bonds to hedge demand risks
  - Butterfly trades
  - May have oscilating price sensitivity to short rate

Localized Demand Effect

What happens when the demand intercept $\theta_0(\tau)$ changes to $\theta_0(\tau) + \Delta\theta_0(\tau)$?

- Demand affects yields through $\int_0^\infty \theta_0(\tau) A_r(\tau) d\tau$ and $\int_0^\infty \theta_0(\tau) A_\beta(\tau) d\tau$
- Long-maturity demand affects demand term more than short rate term, and demand term affects long rate more
- Localized effect of demand change

Calibration

TABLE I

CALIBRATION OF MODEL PARAMETERS FOR THE MAIN SAMPLE OF NOMINAL YIELDS

Parameter	Value	Empirical Moment	Value
κ_r Mean reversion of r_t	0.125	$\sqrt{\text{Var}(y_t^{(1)})}$ Volatility 1-year yield – Levels	2.62
σ_r Diffusion of r_t	0.0146	$\sqrt{\text{Var}(y_{t+1}^{(1)} - y_t^{(1)})}$ Volatility 1-year yield – Annual changes	1.27
κ_β Mean reversion of β_t	0.053	$\frac{1}{30}\sum_{\tau=1}^{30}\sqrt{\text{Var}(y_t^{(\tau)})}$ Volatility τ-year yield – Levels, average over τ	2.20
$a\theta$ Arbitrageur risk aversion × Preferred habitat demand shock	3155	$\frac{1}{30}\sum_{\tau=1}^{30}\sqrt{\text{Var}(y_{t+1}^{(\tau)} - y_t^{(\tau)})}$ Volatility τ-year yield – Annual changes, average over τ	0.796
$a\alpha$ Arbitrageur risk aversion × Preferred habitat demand slope	35.3	$\frac{1}{30}\sum_{\tau=1}^{30}\text{Corr}(y_{t+1}^{(1)} - y_t^{(1)}, y_{t+1}^{(\tau)} - y_t^{(\tau)})$ Correlation 1-year yield with τ-year yield – Annual changes, average over τ	0.504
δ_α Preferred habitat demand shock – short maturities	0.297	$\frac{\sum_{0<\tau\leq 2}\text{Volume}(\tau)}{\sum_{0<\tau\leq 30}\text{Volume}(\tau)}$ Relative volume for maturities $\tau \in (0, 2]$	0.199
δ_θ Preferred habitat demand shock – long maturities	0.307	$\frac{\sum_{11<\tau\leq 30}\text{Volume}(\tau)}{\sum_{0<\tau\leq 30}\text{Volume}(\tau)}$ Relative volume for maturities $\tau \in (11, 30]$	0.094
α Preferred habitat demand slope	5.21	Estimate in KVJ (2012)	−0.746

Policy Experiment: Forward Guidance

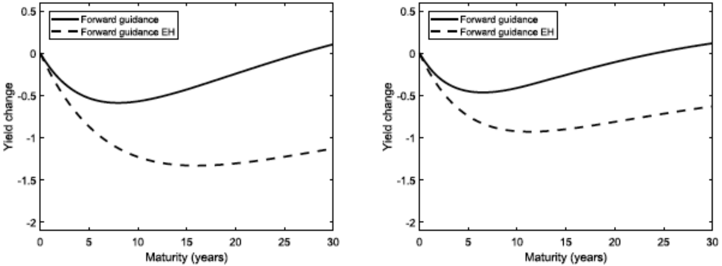


FIGURE 2.—Effect of a forward-guidance announcement about the path of short rates for the calibration based on the main sample of nominal yields.

Policy Experiment: QE

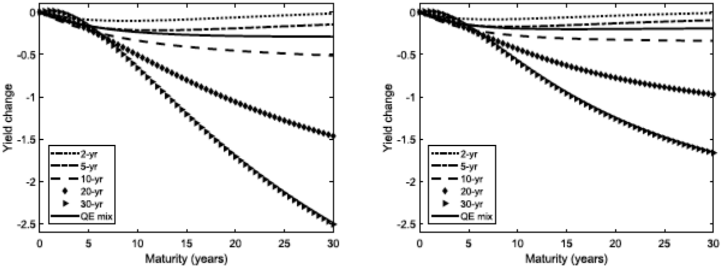


FIGURE 3.—Effect of QE, for the calibration based on the main sample of nominal yields.

Related Literature

- Bond supply effect: Greenwood and Vayanos (2014, RFS)
- Maturity: Guibaud, Nosbusch and Vayanos (2013, RFS)
- Preferred habitat + NK models (Ray, 2019)
- QE + preferred habitat (Droste, Gorodnichenko, Ray, 2024 JPE)
- Preferred habitat models with constrained intermediaries as arbitrageurs (Kekre, Lenel, and Minardi, 2024)
- Preferred habitat model in FX market
    - Gourinchas, Ray and Vayanos (2023)
    - Greenwood, Hanson, Stein and Sundarem (2022, QJE)