The Demand System Approach to Asset Pricing

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No-arbitrage Asset Pricing and Asset Demand

- With no-arbitrage, there exists M such that

$$E(MR) = 1 \text{ for any } R$$

  - Empirical asset pricing: find M, often from the return space
  - The portfolio choice implication of no-arbitrage asset pricing?
- In *traditional* asset pricing, we rarely use asset quantity information mainly due to the limited access to granular holding data
  - Basic economics: prices are determined by demand and supply
  - In asset pricing, where is asset demand and asset supply?
    - Each asset pricing model implies an asset demand function
  - Can we construct asset pricing models that can *jointly fit* asset prices and quantities?

Asset Demand: The Case of CAPM

- N investors, all mean-variance preferences but risk aversions γ_i can be different

$$w_{i,t} = \frac{1}{\gamma_i}\Sigma^{-1}\mu$$

  - Same "objective" belief on μ and Σ
- Implication: all investors hold the same **market** portfolio
- Portfolio construction perspective
  - DeMiguel, Garlappi, and Uppal (2009, RFS): an $1/N$ strategy performs better
  - Estimating μ and Σ is imprecise, the optimal weights are sensitive to the estimates
  - Some early contributions to improve portfolio performance: MacKinlay and Pastor (2000, RFS), Brandt, Santa-Clara, and Valkanov (2009, RFS), Brandt (2009) survey

A Related Literature: Institutional Investors

- The portfolio choice of mutual funds and asset pricing implications
  - De Mirci et al (2022, RFS), Sialm and Zhu (2022, JF), both in international finance
  - Many more on domestic equity and bond markets
- Capital flows into and out of mutual funds
  - Flow-induced trading: Lou (2012, RFS)
  - Flow-performance relation: Berk and Green (2004, JPE)
  - What do mutual fund investors care about: Berk and van Binsbergen (2016, JFE), Barber et al (2016, RFS), Evans and Sun (2021, RFS), Ben-David et al (2022, RFS)
- Not particularly focus on the **equilibrium** asset price determination

Challenge to Studying Prices and Quantities Jointly

- Heterogeneity: Why do different investors choose different portfolios?
  - A joint model of quantity and price is **only** interesting where investors' choices are heterogeneous and the "n-fund" theorem is broken
  - Reasons for heterogeneity: heterogeneous belief, heterogeneous information, heterogeneous preference
- Return moments and characterstics
  - Assets have different characteristics, but investors ultimately care about returns
  - How to relate return moments to characteristics?
- Asset demand not only depends on **the particular** asset's return moment, but also on other assets' return moments, leading to a super high-dimensional problem
  - Require a low-dimensional yet micro-founded asset demand function

Why Do We Want A Joint Model of Prices and Quantities

- Unpack the "black box" of asset price models and make them "tangible"
  - Which investor plays a more important role in a certain market?
  - What will happen to the price of an asset if a certain investor leaves/enters? For example, the effect of quantitative easing?
  - A different approach to link asset price and macro fundamentals
  - ...

The Workshop for the Demand System Approach in May

- Hold every year, introduction to the frontier and hands-on tutorials
- This is a fruitul research area, please register here https://www.koijen.net/index.html
- Here: an introduction to the main methodology, not much into the applications

Outline

1. The demand system approach to asset pricing (Koijen and Yogo, 2019 JPE)
2. Exchange rates and asset prices in global demand system (Koijen and Yogo, 2024)

1. The demand system approach to asset pricing

Koijen and Yogo (2019 JPE)

Overview

1. From optimal portfolio to characteristics-based demand
2. Demand elasticities and market clearing
3. Estimation, identification, and implementation
4. Asset pricing applications

1.1. From optimal portfolio to characteristics-based demand

Notations

- N assets $n = 1, 2, ..., N$
- $S_t(n)$ the number of shares outstanding of asset n at date t
- $P_t(n)$ and $D_t(n)$ the price and dividend of asset n at date t
- Market equity $ME_t(n) = P_t(n)S_t(n)$
- Gross return $R_t(n) = \frac{P_t(n)+D_t(n)}{P_{t-1}(n)}$
- Lower-case variables denote corresponding variables in logs
- Vectors in bold $\mathbf{s}_t = \log(\mathbf{S}_t), \mathbf{p}_t = \log(\mathbf{P}_t), \mathbf{r}_t = \log(\mathbf{R}_t)$
- Characteristics $x_{k,t}(n)$ the k-th characteristic of asset N (K in total), $\mathbf{x}_t$ is a $N \times K$ matrix of characteristics

Where Are We Going

- Start from an optimal portfolio choice problem (micro-founded)
- Derive a demand function that
  - relates asset demand to asset characteristics
  - captures the feature that asset demand depands on **all** assets' returns
  - is low-dimensional and tractable
- The next step: estimation and application

Optimal Portfolio Choice Problem

- I investors, indexed by $i = 1, 2, ..., I$, each with wealth $A_{i,t}$
- Investors have investment universe $\mathcal{N}_{i,t} \subseteq \{1, 2, ., , , .N\}$
  - Investment universe: assets investor i allowed to hold
  - *Why should we have "investment universe" in the model?*
- Investors have log preference (essentially mean-variance)

$$\max_{\mathbf{w}_{i,t}} E_{i,t} \log(A_{i,T})$$

$$s.t. : A_{i,t+1} = A_{i,t} \left(R_{t+1}(0) + \mathbf{w}'_{i,t}(\mathbf{R}_{t+1} - R_{t+1}(0)\mathbf{1}) \right)$$

$$\mathbf{w}_{i,t} \geq 0, \mathbf{1}'\mathbf{w}_{i,t} < 1$$

  - Investors are not allowed to short assets in their universe
  - Investors may choose not to hold some securities whose short-sell constraint binds
  - *Why are short-sale constraints imposed?*

Optimal Portfolio

Denote $\mathbf{w}_{i,t}^{(1)}$ shares of assets with non-binding short-sell constraints

$$\mathbf{w}_{i,t}^{(1)} = \left(\Sigma_{i,t}^{(1,1)}\right)^{-1}\left(\mu_{i,t}^{(1)} - \lambda_{i,t}\mathbf{1}\right)$$

where

$$\mu_{i,t} = E_{i,t}\left[\mathbf{r}_{t+1} - r_{t+1}(0)\mathbf{1}\right] + \frac{\sigma_{i,t}^2}{2}$$

$$\Sigma_{i,t} = E_{i,t}\left[(\mathbf{r}_{t+1} - r_{t+1}(0)\mathbf{1} - E_{i,t}[\mathbf{r}_{t+1} - r_{t+1}(0)\mathbf{1}])(\mathbf{r}_{t+1} - r_{t+1}(0)\mathbf{1})'\right]$$

$\lambda_{i,t}$ is the Lagrangerian multiplier to the constraint $\mathbf{1}'\mathbf{w}_{i,t} < 1$.

- $w_{i,t}(0)$ is the outside asset weight, $w_{i,t}(0) = 1 - \mathbf{1}'\mathbf{w}_{i,t} > 0$
- The expectation is indexed by i - heterogeneous belief

Characteristics and Investors' Information Set

- Denote $\mathbf{x}_t(n)$ the vector of characteristics
  - E.g.: book equity, profitability, investment, market beta
  - Motivated by Fama and French (2015) factor model
- Investors' information set

$$\hat{\mathbf{x}}_{i,t}(n) = \begin{bmatrix} me_t(n) \\ \mathbf{x}_t(n) \\ \log(\varepsilon_{i,t}(n)) \end{bmatrix}$$

  - Include market equity (essentially price)
  - Characteristics $\mathbf{x}_t$ are related to optimal portfolio (explicit later)
  - $\log(\varepsilon_{i,t}(n))$: investors observe, researcher does not
- A technical assumption that leads to logistic specification

$$\mathbf{y}_{i,t}(n) = \begin{bmatrix} \hat{\mathbf{x}}_{i,t}(n) \\ vec(\hat{\mathbf{x}}_{i,t}(n)\hat{\mathbf{x}}_{i,t}(n)') \\ \dots \end{bmatrix}$$

Assumptions

1. Factor structure of returns $\mathbf{r}_{i,t} - r_{i,t}(0) = \Gamma_i f_t + \varepsilon_{i,t}$

$$\Sigma_{i,t} = \Gamma_{i,t}\Gamma_{i,t}^{'} + \gamma_{i,t}\mathbf{I}$$

$$\mu_{i,t} = \Gamma_i \times E_i(f_t)$$

   - $\Gamma_{i,t}$ is a vector of factor loading
   - $\gamma_{i,t}$ is a scalar of idiosyncratic variance

2. Connecting characteristics ($\mathbf{y}_{i,t}$) to factor loadings

$$\mu_{i,t}(n) = \mathbf{y}_{i,t}(n)^{'}\Phi_{i,t} + \phi_{i,t}$$

$$\Gamma_{i,t}(n) = \mathbf{y}_{i,t}(n)^{'}\Psi_{i,t} + \psi_{i,t}$$

- The second assumption requires the asset's own characteristics are sufficient for their factor loadings
- $\mathbf{y}_{i,t}(n)$ is indexed i because the latent demand is included
- Vectors $\Phi_{i,t}, \Psi_{i,t}$ and scalars $\phi_{i,t}, \psi_{i,t}$ common to all assets

Optimal Portfolio and Characteristics-based Asset Demand

Under the two assumptions, the optimal portfolio can be written as

$$w_{i,t}(n) = \mathbf{y}_{i,t}(n)' \Pi_{i,t} + \pi_{i,t}$$

- $\Pi_{i,t}, \pi_{i,t}$ common to all assets
- Investors ultimately care about expected return and covariance. Under these assumptions, characteristics are suffucient.

$$\Pi_{i,t} = \frac{1}{\gamma_{i,t}} \left(\Phi_{i,t} - \Psi_{i,t}\kappa_{i,t}\right), \pi_{i,t} = \frac{1}{\gamma_{i,t}} \left(\phi_{i,t} - \lambda_{i,t} - \psi_{i,t}\kappa_{i,t}\right)$$

- Investors prefer assets with characteristics that lead to higher expected returns, lower loading on systematic factors, and lower idiosyncratic variance
- $\lambda_{i,t}$ and $\kappa_{i,t}$ depend on characteristics of **all** assets

Demand Specification

- Portfolio share of asset n

$$w_{i,t}(n) = \frac{\delta_{i,t}(n)}{1 + \sum_{m \in \mathcal{N}_{i,t}} \delta_{i,t}(m)}$$

- Portfolio share of the outside asset 0

$$w_{i,t}(0) = \frac{1}{1 + \sum_{m \in \mathcal{N}_{i,t}} \delta_{i,t}(m)}$$

- Characteristic-based demand

$$\frac{w_{i,t}(n)}{w_{i,t}(0)} \equiv \delta_{i,t}(n) = \exp\left\{\beta_{0,i,t} me_t(n) + \sum_{k=1}^{K-1} \beta_{k,i,t} x_{k,t}(n) + \beta_{K,i,t}\right\} \varepsilon_{i,t}(n)$$

Normalize $\varepsilon_{i,t}(n)$ to have unity average to identify $\beta_{K,i,t}$

  - Derivation skipped here, see Appendix A, Proof of Corollary 1

Summary

- Micro-founded asset demand
  - Capture the feature that asset demand depends on characteristics of **all** assets
  - Based on the assumption of the factor structure of asset returns and the postulated relation between characteristics and factor loadings
  - Sparse in terms of the demand specification
- Estimable using data on price, quantity and characteristics

1.2. Demand elasticities and market clearing

What's Next

- Up to now: a characteristic-based asset demand function
- Next: what does this model say about the demand elasticity
  - for each investor i
  - for the aggregate market

Demand Elasticities

- An important object of interest is demand elasticity, i.e., how much will demand change if asset price changes by 1 percent
- The vector of shares $\mathbf{q}_{i,t}$

$$\mathbf{q}_{i,t} = \log(A_{i,t}\mathbf{w}_{i,t}) - \mathbf{p}_t$$

- Demand elasticity

$$-\frac{\partial \mathbf{q}_{i,t}}{\partial \mathbf{p}_t'} = \mathbf{I} - \beta_{0,i,t} diag(\mathbf{w}_{i,t})^{-1} \mathbf{G}_{i,t}$$

  where $\mathbf{G}_{i,t} = diag(\mathbf{w}_{i,t}) - \mathbf{w}_{i,t}\mathbf{w}_{i,t}'$.
- Required assumption: $\beta_{0,i,t} < 1$

Derivation (1)

- Express demand elasticity as function of δ elasticity

$$-\frac{\partial \mathbf{q}_{i,t}}{\partial \mathbf{p}_t'} = I - \frac{\partial \log \mathbf{w}_{i,t}}{\partial \mathbf{p}_t'}$$

- Define $\bar{\delta}_{i,t} = \sum_{n=1}^{N} \delta_{i,t}(n)$. For a typical asset l, m:

$$\frac{\partial \log w_{i,t}(l)}{\partial p_t(l)} = \frac{\delta_{i,t}(l)}{w_{i,t}(l)(1+\bar{\delta}_{i,t})}\left[\frac{\partial \delta_{i,t}/\partial p_{i,t}(l)}{\delta_{i,t}(l)} - \frac{\partial \bar{\delta}_{i,t}/\partial p_t(l)}{1+\bar{\delta}_{i,t}}\right]$$

$$= \frac{\partial \delta_{i,t}(l)/\partial p_{i,t}(l)}{\delta_{i,t}(l)} - \frac{\partial \bar{\delta}_{i,t}/\partial p_t(l)}{1+\bar{\delta}_{i,t}}$$

$$\frac{\partial \log w_{i,t}(m)}{\partial p_t(l)} = -\frac{\partial \bar{\delta}_{i,t}/\partial p_t(l)}{1+\bar{\delta}_{i,t}} \text{ for } m \neq l$$

Derivation (2)

- Partial derivative of δ's with respect to price

$$\frac{\partial \delta_{i,t}(l)/\partial p_{i,t}(l)}{\delta_{i,t}(l)} = \beta_{0,i,t}$$

$$\frac{\partial \bar{\delta}_{i,t}/\partial p_t(l)}{1+\bar{\delta}_{i,t}} = \frac{\beta_{0,i,t}}{1+\bar{\delta}_{i,t}}\delta_{i,t}(l) = \beta_{0,i,t} w_{i,t}(l)$$

- Plug into the expression of $\frac{\partial q_{i,t}}{\partial p_t^{'}}$

$$\begin{bmatrix} -\frac{\partial q_{i,t}(1)}{\partial p_t(1)} & \dots & -\frac{\partial q_{i,t}(1)}{\partial p_t(N)} \\ \dots & \dots & \dots \\ -\frac{\partial q_{i,t}(N)}{\partial p_t(1)} & \dots & -\frac{\partial q_{i,t}(N)}{\partial p_t(N)} \end{bmatrix} = I - \beta_{0,i,t} I + \beta_{0,i,t} \begin{bmatrix} w_{i,t}(1) & \dots & w_{i,t}(N) \\ \dots & \dots & \dots \\ w_{i,t}(1) & \dots & w_{i,t}(N) \end{bmatrix}$$

This expression is equivalent to the one previously shown:

$$\begin{bmatrix} w_{i,t}(1) & \dots & w_{i,t}(N) \\ \dots & \dots & \dots \\ w_{i,t}(1) & \dots & w_{i,t}(N) \end{bmatrix} = diag(\mathbf{w}_{i,t})^{-1}\mathbf{w}_{i,t}\mathbf{w}_{i,t}^{'}$$

Remark on Cross-Elasticity

- The demamd system imposes cross-asset dependence via market clearing
  - If asset *l*'s price increases by 1 percent, its **relative demand** decreases by $1 - \beta_{0,i,t}$
  - Since portfolio weights have to add up to one, all other assets' weight increases - as a result, the weight for asset *l* decreases by less than $1 - \beta_{0,i,t}$

Aggregate Demand Elasticity

Define $\mathbf{q}_t = \log\left(\sum_{i=1}^{I} A_{i,t}\mathbf{w}_{i,t}\right) - \mathbf{p}_t$, aggregate demand elasticity

$$-\frac{\mathbf{q}_t}{\mathbf{p}_t^{'}} = I - \sum_{i=1}^{I} \beta_{0,i,t} A_{i,t} \mathbf{H}_t^{-1} \mathbf{G}_{i,t}$$

where $\mathbf{H}_t = \sum_{i=1}^{I} A_{i,t} diag(\mathbf{w}_{i,t})$.

Derivation

- For typical asset l and m

$$\frac{\partial q_t(l)}{\partial p_t(l)} = -1 + \frac{\sum_{i=1}^{I} A_{i,t} \partial w_{i,t}(l) / \partial p_t(l)}{\sum_{i=1}^{I} A_{i,t} w_{i,t}(l)} = -1 + \frac{\sum_{i=1}^{I} A_{i,t} w_{i,t}(l) \beta_{0,i,t} [1 - w_{i,t}(l)]}{\sum_{i=1}^{I} A_{i,t} w_{i,t}(l)}$$

$$\frac{\partial q_t(m)}{\partial p_t(l)} = -1 + \frac{\sum_{i=1}^{I} A_{i,t} \partial w_{i,t}(m) / \partial p_t(l)}{\sum_{i=1}^{I} A_{i,t} w_{i,t}(m)} = -1 - \frac{\sum_{i=1}^{I} A_{i,t} w_{i,t}(m) \beta_{0,i,t} w_{i,t}(l)}{\sum_{i=1}^{I} A_{i,t} w_{i,t}(m)}$$

In matrix form, it is equivalent to the expression before.

Market Clearing and Asset Price Determination

- Market clearing condition for asset n

$$ME_t(n) = \sum_{i=1}^{I} A_{i,t} w_{i,t}(n)$$

Rewrite in logarithm

$$\mathbf{p} = f(\mathbf{p}) = \log\left(\sum_{i=1}^{I} A_t \mathbf{w}_i(\mathbf{p})\right) - \mathbf{s}$$

- Again, notice that $w_i(l)$ not only depends on $p(l)$, but also all other asset prices
- However, $\delta_i(n) \equiv \frac{w_i(n)}{w_i(0)}$ only depends on $p(n)$
- This is a **system** of N decoupled nonlinear equations
- The solution of the system $\mathbf{p}$: the equilibrium asset price vector

Computing $\mathbf{p}$

- Newton's method. Start with p_m, iterate through

$$\mathbf{p}_{m+1} = \mathbf{p}_m + \left(I - \frac{\partial f(\mathbf{p}_m)}{\partial \mathbf{p}_m'} \right)^{-1} (f(\mathbf{p}_m) - \mathbf{p}_m)$$

- Challenge: the derivative is high-dimensional
- Solution: approximate the derivative using diagonals only

$$\frac{\partial f(\mathbf{p}_m)}{\partial \mathbf{p}_m'} \approx diag\left(min\left\{ \frac{\sum_{i=1}^{I} \beta_{0,i} A_i w_i(\mathbf{p}_m; n)(1 - w_i(\mathbf{p}_m; n))}{\sum_{i=1}^{I} A_i w_i(\mathbf{p}_m; n)} \right\}, 0 \right)$$

Different Asset Pricing Approaches

- No-arbitrage (traditional asset pricing)
  - Asset returns only
- Dynamic equilibrium (macro-finance)
  - Micro-founded no-arbitrage
  - Complete market: separate price and quantity
  - Incomplete market
    - Price and quantity constitute a fixed-point problem: asset demand depends on perceived asset return distribution, return distribution depends on the portfolio choice
    - Dynamic consistency: perceived distribution consistent with the ex post realized return distribution
- Demand system approach
  - Model demand function incorporating rich heterogeneity
  - Drop the dynamic consistency requirement in the dynamic equilibrium approach
  - Retain the basic portfolio choice structure and framework

1.3. Estimation, identification, and implementation

What's Next

- Up to now: characteristics-based asset demand and the implied demand elasticity for investor i and the aggregate market
- Next: how to estimate demand function?

Challenges for Estimating Demand for US Stocks

- A key feature of US stock holding data: sparsity of holding
  - Challenge: how to deal with zeros
  - Define investment universe: the set of securities investors are allowed to choose from
  - Nonlinear GMM
- Challenge: the endogeneity issue
  - The asset price ($me(n)$) is, by construction, endogenous and correlated with $\varepsilon_{i,t}(n)$
  - Require an instrument

Data

- Prices and characteristics data are standard
  - Log book equity, profitability, investment, dividend to book equity, market beta, etc
  - Not include return variables - assume all characteristics except for *me* are exogenous
- Holding data: Thomson Reuters Institutional Holdings Database, compied from quarterly filings of SEC Form 13F
  - All institutional investment managers that exercise investment discretion on accounts holding Section 13(f) securities exceeding $100 million in total market value must file the form
  - **Only long positions**, no info on cash and bond positions (not 13(f) securities)

Data

- Institutions: Banks, insurance companies, investment advisors, mutual funds, pension funds, and other 13F institutions
- Value: price $\times$ shares held
- Portfolio share: value/total AUM
- The gap: outside asset

Data

- Investment universe for an institution: stocks that are currently held or ever held in the previous 11 quarters
  - Why 11 quarters? See next slide
- Shares outstanding equals shares held by all investors
  - Data not covering all investors
  - Define households as the residual (including household holdings and small institutions that do not have to file Form 13F)

Investment Universe: Persistence of Holding

Table 1
Persistence of the Set of Stocks Held

AUM	Previous quarters										
percentile	1	2	3	4	5	6	7	8	9	10	11
1	82	85	86	88	89	90	91	92	93	93	94
2	85	87	89	91	92	92	93	94	94	95	95
3	85	88	89	90	91	92	93	93	94	94	95
4	85	87	89	90	91	92	92	93	93	94	94
5	85	87	89	90	90	91	92	92	93	93	94
6	85	87	88	89	90	91	92	92	93	93	94
7	84	86	88	89	90	91	91	92	92	93	93
8	84	87	88	90	90	91	92	92	93	93	94
9	87	89	90	91	92	93	93	94	94	94	95
10	92	93	94	95	95	96	96	96	97	97	97

Note.—This table reports the percentage of stocks held in the current quarter that were ever held in the previous one to eleven quarters. Each cell is a pooled median across time and all institutions in the given assets under management (AUM) percentile. The quarterly sample period is from 1980:1 to 2017:4.

Latent Demand and the Endogeneity Issue

- Latent demand $\varepsilon_{i,t}(n)$ is observed by investor i, but not by the econometrician
- Directly estimating the regression has endogeneity issue, $me_t(n)$ is correlated with unobservable demand shocks $\varepsilon_{i,t}(n)$
- A standard issue in demand estimation (classic example of simultaneity bias in econometrics), need an instrument

Identifying Assumptions

Directly estimating the demand equation requires

$$E[\varepsilon_i(n)|me(n), x(n)] = 1$$

- Following the literature, treat characteristics other than prices as exogenous
- Some argue that investors are price takers and individual demand shocks $\varepsilon_i(n)$ do not have price impact - not right if demand shocks are correlated
- Seek for an instrument for $me(n)$, to be specified later so that

$$E[\varepsilon_i(n)|\hat{me}(n), x(n)] = 1$$

- Assume wealth distribution predetermined and exogenous to demand shocks

Asset Demand with Investment Mandate

$$\delta_i(n) = \exp\left\{\beta_{0,i} me(n) + \sum_{k=1}^{K-1} \beta_{k,i} x_k(n) + \beta_{K,i}\right\} \varepsilon_i(n) \text{ if } n \in \mathcal{N}_i$$

Otherwise, $\delta_i(n) = 0$.

- An investor does not hold an asset for two reasons: not allowed (not in the investment universe), or chooses not to (in the investment universe)

Instrument Construction

$$\hat{me}_i(n) = \log\left(\sum_{j\neq i} A_j \frac{\mathbb{I}_j(n)}{1+\sum_{m=1}^{N}\mathbb{I}_j(m)}\right)$$

- The idea of constructing the instrument: If an asset n is in the investment universe of many other institutions (i' own universe may be endogenous), its demand is likely to be high and price likely to be high
- *Why scaled by how many assets in the universe?* The larger the numer of assets in the universe, the smaller allocated to n (assuming $1/N$ strategy)
- Exploit variation in the investment universe across investors and the size of potential investors across assets
- Identification assumption is satisfied

$$E[\varepsilon_i(n)|\hat{me}(n), x(n)] = 1$$

Log-linear Regression and Nonlinear GMM

- Log-linear regression

$$\log \delta_i(n) = \beta_{0,i} me(n) + \sum_{k=1}^{K-1} \beta_{k,i} x_k(n) + \beta_{K,i} + \tilde{\varepsilon}_i(n)$$

  - Identifying assumption
$$E[\tilde{\varepsilon}_i(n) | \hat{me}(n), x(n)] = 0$$
  - Have to drop all zeros, even if it is in the investment universe
  - Not appropriate if we think zeros in the investment universe are driven by short-sale constraint, i.e., $\tilde{\varepsilon}_i(n) = 0$
  - Depending on how we think about zeros - include zeros in the regression?

IV Validity

- First-stage weak IV test
- IV exogeneity? Depending on the exogeneity of the wealth distribution of other investors A_j, and the exogeneity of investment universe of other investors
  - Investment universe is very persistent and does not vary much with time
  - Lend support to a predetermined investment universe

Sparsity of Portfolio Holding

Table 2
Summary of 13F Institutions

Period	Number of institutions	% of market held	Assets under management ($ million): Median	Assets under management ($ million): 90th percentile	Number of stocks held: Median	Number of stocks held: 90th percentile	Number of stocks in investment universe: Median	Number of stocks in investment universe: 90th percentile
1980–1984	544	35	337	2,666	118	386	183	523
1985–1989	780	41	400	3,604	116	451	208	692
1990–1994	979	46	405	4,566	106	512	192	811
1995–1999	1,319	51	465	6,579	102	556	176	943
2000–2004	1,800	57	371	6,095	88	521	165	983
2005–2009	2,442	65	333	5,427	73	460	145	923
2010–2014	2,879	65	315	5,441	68	447	122	800
2015–2017	3,655	68	302	5,204	67	454	112	748

Note.—This table reports the time-series mean of each summary statistic within the given period, based on Securities and Exchange Commission Form 13F. The quarterly sample period is from 1980:1 to 2017:4.

Pooling Estimation

- Each institution's holding is too small to accurately estimate the demand equation for individual investors
- Estimate by institution when there are more than 1000 strictly positive holdings
- Pool similar institutions that have smaller than 1000 positive holdings
- Estimate cross-section by cross-section

A Remark on Estimation

Here we ignore the problems of zeros and think about the log-linear specification.

- Cross-sectional (different securities) regression for each i
  - Same result with $\delta_{i,t}(n)$ or $\ln w_{i,t}(n)$ on the LHS, as $\ln w_{i,t}(0)$ is absorbed by the intercept. However, when calculating the elasticity, we need to take into account the effect of price change on $w_{i,t}(0)$
- Cross-sectional regression with pooled i
  - Assume they have the same demand function
  - The only difference across different investors: $\varepsilon_{i,t}(n)$
  - Different investors have different $w_{i,t}(0)$ - using $\delta_{i,t}(n)$ or $w_{i,t}(n)$ on the LHS lead to the same result **only when** i-FEs are included
  - In a cross-sectional regression with pooled i, since we assume all investors have the same belief so that they should have the same $\lambda_{i,t}$ and $\kappa_{i,t}$ that depends on a characteristics of **all** assets, so not necessary to include i-FE
  - Adding an i- FE: assuming an ad-hoc higher demand (expected return) for some investors than others
- Panel regression (later)
  - The intercepts and regression coefficients are time-varying - left for the next paper

Coefficients on Characteristics

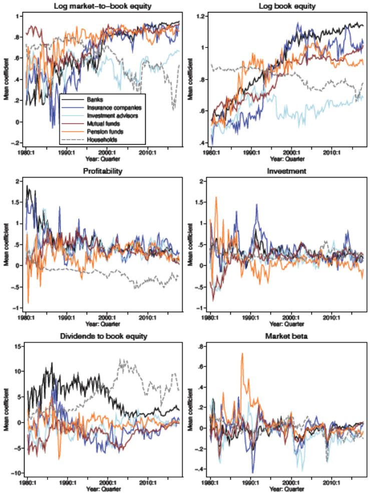

Figure 3. Coefficients on characteristics. Characteristics-based demand (10) is estimated for each institution at each date by GMM under moment condition (20). This figure reports the cross-sectional mean of the estimated coefficients by institution type, weighted by assets under management. The quarterly sample period is from 1980:1 to 2017:4.

Demand Elasticity

- A lower coefficient on *me* implies a higher demand elasticity
  - $\beta_{0,i} < 1$, LHS is portfolio share, calculated using price $\times$ quantity
  - The effect of p on q should substract 1, and lower the coefficient, the more demand drops with higher price
- Mutual funds have less elastic demand than other types of institutions or households
- Banks, insurance companies, and pension funds have become less elastic over time, while households become more elastic

Demand of Other Characteristics

- Banks and insurance companies tilt portfolios toward largers stocks, while investment advisors tilt portfolios toward smaller stocks
- Investment advisors prefer stocks with lower *me*, higher profitability, lower investment, and lower market beta than households
  - These are characteristics related to positive returns
  - "Smart money" investors
- The demand for market beta falls in recessions

Cross-sectional Dispersion of Latent Demand

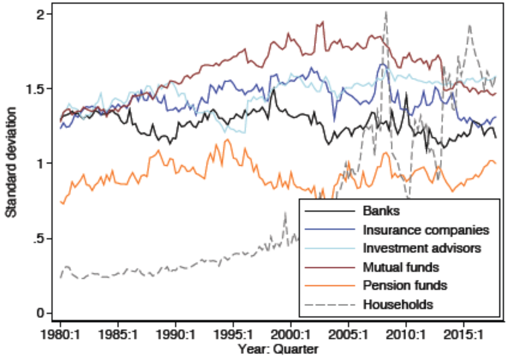

Figure 4. Standard deviation of latent demand. Characteristics-based demand (10) is estimated for each institution at each date by GMM under moment condition (20). This figure reports the cross-sectional standard deviation of log latent demand by institution type, weighted by assets under management. The quarterly sample period is from 1980:1 to 2017:4.

1.4. Asset pricing applications

Application 1: Price Impact of Demand Shocks

The impact of investor i's latent demand on the **equilibrium** asset price

$$\frac{\partial \mathbf{p}_t}{\partial \log(\varepsilon_{i,t})'} = \left(I - \sum_{j=1}^{I} A_{j,t}\beta_{0,j,t}H_t^{-1}G_{j,t} \right)^{-1} A_{i,t}H_t^{-1}G_{i,t}$$

- Derivation skipped
- The idea: investor i increases the demand for asset n and increases asset n's price
- All other investors reduce their demand for asset n
- Find the new equilibrium price when the increased demand for i equals the decreased demand for other investors

Results

- With downward-sloped demand, demand shocks have persistent effect on prices

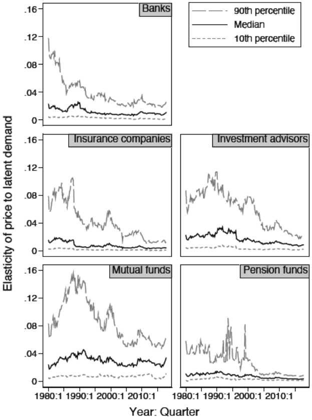

Figure 5. Price impact across stocks and institutions. Price impact for each stock and institution is estimated through the diagonal elements of matrix (23), then averaged by institution type. This figure summarizes the cross-sectional distribution of price impact across stocks for the average bank, insurance company, investment advisor, mutual fund, and pension fund. The quarterly sample period is from 1980:1 to 2017:4.

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Application 2: Variance Decomposition of Stock Returns

$$r_{t+1} = p_{t+1} - p_t + v_{t+1}$$

where $v_{t+1} = \log(1 + \exp(d_{t+1} - p_{t+1}))$. Define

$$p_{t+1} - p_t = \Delta p_{t+1}(s) + \Delta p_{t+1}(x) + \Delta p_{t+1}(A) + \Delta p_{t+1}(\beta) + \Delta p_{t+1}(\varepsilon)$$

And the Δ's are defined recursively as

$$\Delta p_{t+1}(s) = g(s_{t+1}, x_t, A_t, \beta_t, \varepsilon_t) - g(s_t, x_t, A_t, \beta_t, \varepsilon_t)$$

$$\Delta p_{t+1}(x) = g(s_{t+1}, x_{t+1}, A_t, \beta_t, \varepsilon_t) - g(s_{t+1}, s_t, A_t, \beta_t, \varepsilon_t)$$

$$\dots$$

$$var(r_{t+1}) = cov(\Delta p_{t+1}(s), r_{t+1}) + cov(\Delta p_{t+1}(x), r_{t+1}) + cov(v_{t+1}, r_{t+1})$$
$$+cov(\Delta p_{t+1}(A), r_{t+1}) + cov(\Delta p_{t+1}(\beta), r_{t+1}) + cov(\Delta p_{t+1}(\varepsilon), r_{t+1})$$

Variance Decomposition

TABLE 3
VARIANCE DECOMPOSITION OF STOCK RETURNS

	% of variance
Supply:	
Shares outstanding	2.1
	(0.2)
Stock characteristics	9.7
	(0.3)
Dividend yield	0.4
	(0.0)
Demand:	
Assets under management	2.3
	(0.1)
Coefficients on characteristics	4.7
	(0.2)
Latent demand: Extensive margin	23.3
	(0.3)
Latent demand: Intensive margin	57.5
	(0.4)
Observations	134,328

Note.—The cross-sectional variance of annual stock returns is decomposed into supply- and demand-side effects. Heteroskedasticity-robust standard errors are reported in parentheses. The annual sample period is from 1981 to 2017.

Application 3: Stock Market Volatility in 2008

- Do large institutional investors amplify volatility in bad times?
- A modified variance decomposition

$$var(r_{t+1}) = cov(\Delta p_{t+1}(s) + \Delta p_{t+1}(x) + v_{t+1}, r_{t+1})$$

$$+\sum_{i=1}^{I} cov(\Delta p_{t+1}(A_i) + \Delta p_{t+1}(\beta_i) + \Delta p_{t+1}(\varepsilon_i), r_{t+1})$$

Variance Decomposition: 2007-2008

TABLE 4
VARIANCE DECOMPOSITION OF STOCK RETURNS IN 2008

AUM ranking	Institution	AUM ($ billion)	Change in AUM (%)	% of variance	
	Supply: Shares outstanding, stock characteristics & dividend yield			8.1	(1.0)
1	Barclays Bank	699	-41	0.3	(0.1)
2	Fidelity Management & Research	577	-63	0.9	(0.2)
3	State Street Corporation	547	-37	0.3	(0.0)
4	Vanguard Group	486	-41	0.4	(0.0)
5	AXA Financial	309	-70	0.3	(0.1)
6	Capital World Investors	309	-44	0.1	(0.1)
7	Wellington Management Company	272	-51	0.4	(0.1)
8	Capital Research Global Investors	270	-53	0.1	(0.1)
9	T. Rowe Price Associates	233	-44	-0.2	(0.1)
10	Goldman Sachs & Company	182	-59	0.1	(0.1)
11	Northern Trust Corporation	180	-46	0.1	(0.0)
12	Bank of America Corporation	159	-50	0.0	(0.1)
13	J.P Morgan Chase & Company	153	-51	0.1	(0.1)
14	Deutsche Bank	136	-86	0.3	(0.1)
15	Franklin Resources	135	-60	0.2	(0.1)
16	College Retire Equities	135	-55	0.0	(0.0)
17	Janus Capital Management	134	-53	0.3	(0.1)
18	MSDW & Company	133	45	0.1	(0.1)
19	Amvescap London	110	-42	0.0	(0.1)
20	Dodge & Company	93	-65	0.0	(0.0)
21	UBS Global Asset Management	90	-63	0.0	(0.1)
22	Davis Selected Advisers	87	-54	0.0	(0.0)
23	Neuberger Berman	86	-73	0.0	(0.1)
24	Blackrock Investment Management	86	-69	0.0	(0.0)
25	OppenheimerFunds	83	-64	0.2	(0.1)
26	Wells Fargo & Norwest Corporation	75	-56	0.1	(0.1)
27	MFS Investment Management	73	-44	0.0	(0.0)
28	Putnam Investment Management	73	-76	0.1	(0.1)
29	Marsico Capital Management	73	-56	0.0	(0.0)
30	Lord, Abbett & Company	72	-61	0.3	(0.1)
	Subtotal: 30 largest institutions	6,050	-48	4.4	
	Smaller institutions	6,127	-53	40.7	(2.3)
	Households	6,322	-47	46.9	(2.6)
	Total	18,499	-49	100.0	

Note.—The cross-sectional variance of annual stock returns in 2008 is decomposed into supply- and demand-side effects. This table reports the total demand-side effect for each institution due to changes in assets under management (AUM), the coefficients on characteristics, and latent demand. The largest 30 institutions are ranked by AUM in 2007:4. Heteroskedasticity-robust standard errors are reported in parentheses.

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Application 4: Return Predictability

$$p_T = g(s_T, x_T, A_T, \beta_T, \varepsilon_T)$$

- Up to the first-order, we can rewrite long-run capital gain as

$$E_t[p_T - p_t] = g(E_t s_T, E_t x_T, E_t A_T, E_t \beta_T, E_t \varepsilon_T) - p_t$$

- If any characteristic is predictable, return is predictable
- Assume random walk for characteristics and $E_t \varepsilon_T = 1$, so that the long-run expected return

$$E_t[p_T - p_t] = g(s_t, x_t, A_t, \beta_t, 1) - p_t$$

- Control for other known sources of predictability, test the long-run return predictability from the mean reversion of latent demand (in the cross-section)

Return Predictability

TABLE 5
RELATION BETWEEN STOCK RETURNS AND CHARACTERISTICS

Characteristic	All stocks	Excluding microcaps
Expected return	0.18	0.11
	(0.04)	(0.04)
Log market equity	-0.25	-0.15
	(0.08)	(0.08)
Book-to-market equity	0.04	0.06
	(0.04)	(0.05)
Profitability	0.30	0.29
	(0.06)	(0.06)
Investment	-0.38	-0.21
	(0.03)	(0.03)
Market beta	0.08	0.01
	(0.08)	(0.10)
Momentum	0.24	0.37
	(0.08)	(0.10)

Note.—Monthly excess returns, over the 1-month T-bill rate, are regressed onto lagged characteristics. This table reports the time-series mean and standard errors of the estimated coefficients. Microcaps are stocks whose market equity is below the 20th percentile for NYSE stocks. The monthly sample period is from June 1980 to December 2017.

2. Exchange rates and asset prices in global demand system

Koijen and Yogo (2024)

What Does This Paper Do?

- The determination of international asset prices, including long-term bond yields, stock price, and exchange rates
- A global asset demand system: 37 countries
  - Investor at the country level
  - Assets (long-term bond, short-term bond, and equity) at country level (88 countries)

Overview

1. A simple model for demand function
2. Characteristics-based demand system
3. Data, instrument and estimation result
4. Applications

2.1. A simple model for demand function

A 2-Period, 2-Country Model

- Asset markets
- Investors
- Consumption and portfolio choice
- Short-term rate model
- Asset demand
- Market clearing

Asset Markets

- Two countries: US and Japan
- Price index $B_{U,t}, B_{J,t}$, define $V_t = B_{U,t}/B_{J,t}$ and real exchange rate E_t/V_t
- Each country has a riskless bond in respective local currency $P_t(U), P_t(J)$; face value $Q_t(U), Q_t(J)$, lower-case letters to indicate logs

Investors: US

$$\max_{C_{U,t}, w_{U,t}(J)} \frac{C_{U,t}^{1-\gamma}}{1-\gamma} + \beta \frac{E_{U,t}[C_{U,t+1}^{1-\gamma}]}{1-\gamma}$$

- Wealth $A_{U,t} = A_{U,t-} + Y_{U,t} - B_{U,t} C_{U,t}$
- Real return on assets

$$R_{U,t+1} = \left(R_{t+1}(U) + w_{U,t}(J) \left(\frac{R_{t+1}(J) E_{t+1}}{E_t} - R_{t+1}(U) \right) \right) \frac{B_{U,t}}{B_{U,t+1}}$$

- Terminal consumption $C_{U,t+1} = A_{U,t} R_{U,t+1}$
- Euler equations

$$E_{U,t} \left[\beta \left(\frac{C_{U,t+1}}{C_{U,t}} \right)^{-\gamma} \frac{R_{t+1}(U) B_{U,t}}{B_{U,t+1}} \right] = 1$$

$$E_{U,t} \left[\beta \left(\frac{C_{U,t+1}}{C_{U,t}} \right)^{-\gamma} \frac{R_{t+1}(J) E_{t+1} B_{U,t}}{E_t B_{U,t+1}} \right] = 1$$

Investors: Japan

Analogous to US investors. Notice that $A_{J,t}$ is denominated in USD

$$\max_{C_{J,t}, w_{J,t}(U)} \frac{C_{J,t}^{1-\gamma}}{1-\gamma} + \beta \frac{E_{J,t}[C_{J,t+1}^{1-\gamma}]}{1-\gamma}$$

- Wealth $A_{J,t} = A_{J,t-} + E_t(Y_{J,t} - B_{J,t} C_{J,t})$
- Real return on assets

$$R_{J,t+1} = \left(R_{t+1}(J) + w_{J,t}(U) \left(\frac{R_{t+1}(U) E_t}{E_{t+1}} - R_{t+1}(J) \right) \right) \frac{B_{J,t}}{B_{J,t+1}}$$

- Terminal consumption $C_{J,t+1} = \frac{A_{J,t}}{E_t} R_{J,t+1}$
- Euler equations

$$E_{J,t} \left[\beta \left(\frac{C_{J,t+1}}{C_{J,t}} \right)^{-\gamma} \frac{R_{t+1}(J) B_{J,t}}{B_{J,t+1}} \right] = 1$$

$$E_{J,t} \left[\beta \left(\frac{C_{J,t+1}}{C_{J,t}} \right)^{-\gamma} \frac{R_{t+1}(U) E_t B_{J,t}}{E_{t+1} B_{J,t+1}} \right] = 1$$

Consumption and Optimal Portfolio

- Denote

$$\mu_{U,t}(J) = E_{U,t}\left[r_{t+1}(J) + \Delta e_{t+1} - r_{t+1}(U)\right]$$
$$\sigma^2_{U,t}(J) = var_{U,t}\left[r_{t+1}(J) + \Delta e_{t+1} - \Delta b_{U,t+1}\right]$$
$$\mu_{J,t}(U) = E_{J,t}\left[r_{t+1}(U) - \Delta e_{t+1} - r_{t+1}(J)\right]$$
$$\sigma^2_{J,t}(U) = var_{J,t}\left[r_{t+1}(U) - \Delta e_{t+1} - \Delta b_{J,t+1}\right]$$

  Note: Allow for different expectations for different investors

- Optimal portfolio $n \neq i$

$$w_{i,t}(n) = \frac{\mu_{i,t}(n) + \sigma^2_{i,t}(n)/2}{\gamma \sigma^2_{i,t}(n)}$$

Short-term Rate Model

Let $\mathbf{z}_t(n)$ be a vector of macro variables of country n

$$p_t(n) = \Pi^{'} \mathbf{z}_t(n) + \zeta_t(n)$$

Asset Demand

- Model expected return

$$\mu_{U,t}(J) = p_t(U) - p_t(J) - \Theta(e_t - v_t)$$

$$\mu_{J,t}(U) = p_t(J) - p_t(U) + \Theta(e_t - v_t)$$

$$\sigma^2_{i,t}(n) = \exp(-\Psi' \mathbf{x}_{i,t}(n) - \psi_{i,t}(n))$$

  - International bond returns have a factor structure and loadings depend on aset characteristics

- Portfolio weight

$$w_{i,t}(n) = \frac{\mu_{i,t}(n) \exp(\Psi' \mathbf{x}_{i,t}(n) + \psi_{i,t}(n)) + 1/2}{\gamma}$$

Market Clearing

- Goods market clearing

$$Y_{U,t} + E_t Y_{J,t} = B_{U,t} C_{U,t} + E_t B_{J,t} C_{J,t}$$

  - *Should we have two market clearing conditions?* - one good, two price levels
- Balance of payment

$$A_{U,t} - A_{U,t-} + A_{J,t} - A_{J,t-} = 0$$

- Asset market clearing

$$P_t(n) Q_t(n) = A_{U,t} w_{U,t}(n) + A_{J,t} w_{J,t}(n)$$

There are four market clearing conditions that determine three asset prices, $P_t(U), P_t(J), E_t$.

2.2. Characteristics-based demand system

Notation

- N issuer countries, $n = 1, 2, .., N$
- Three asseet classes: short-term debt ($l = 1$), long-term debt ($l = 2$), equity $l = 3$
- $P_t(n, l)$ the market-to-book ratio for asset class l in country n at time t (price)
- $Q_t(n, l)$ the total book value in country n's currency unit of asset class l in country n at time t (quantity)
- $E_t(n)$ the nominal exchange rate in USD per currency n

A Demand System Based Asset Pricing Model

- Expected return modeling
- Characteristic-based demand
  - Within asset class
  - Across asset class
- Market clearing

Expected Return Modelling

- Similar to KY (2019), characteristics affect expected return
- Model expected returns

$$r_{t+1}(n,l) - y_t(US) = \theta_l mb_t(n,l) + \Theta_l(e_t(n) - z_t(n)) + \chi_{n,l} + \nu_{t+1}(n,l)$$

  - Use the *mb* and real exchange rate as return predictor to measure expected returns
  - Expected excess return (in LC) expressed as

$$\mu_{i,t}(n,l) = \theta_l p_t(n,l) + \Theta_l(e_t(n) - z_t(n)) - \theta_1 p_t(i,1) - \Theta_1(e_t(i) - z_t(i))$$

- Assumption: investors care about returns denominated in their residence currency

Characteristics-based Demand: Within Asset Class

$$w_{i,t}(n,l) = w_{i,t}(l) w_{i,t}(n|l)$$

where

$$w_{i,t}(n|l) = \frac{\delta_{i,t}(n,l)}{1 + \sum_{m=1}^{N} \delta_{i,t}(m,l)}$$

and

$$\log(\delta_{i,t}(n,l)) = \lambda_l \mu_{i,t}(n,l) + \Lambda_l^{'} x_{i,t}(n) + \varepsilon_{i,t}(n,l)$$

$$w_{i,t}(0|l) = \frac{1}{1 + \sum_{m=1}^{N} \delta_{i,t}(m,l)}$$

Note that characteristics not only depends on n but also on i, which allows for bilateral characteristics

Characteristics-based Demand: Across Asset Class

Specify the portfolio weight into asset class $w_{i,t}^{l}$ as

$$w_{i,t}(l) = \frac{(1 + \sum_{m=1}^{N} \delta_{i,t}(m,l))^{\rho_l} \exp(\alpha_l + \xi_{i,t}(l))}{\sum_{k=1}^{3}(1 + \sum_{m=1}^{N} \delta_{i,t}(m,k))^{\rho_k} \exp(\alpha_k + \xi_{i,t}(k))}$$

Estimate the following regression

$$\log\left(\frac{w_{i,t}(l)}{w_{i,t}(3)}\right) = -\rho_l \log(w_{i,t}(0|l)) + \rho_3 \log(w_{i,t}(0|3)) + \alpha_l + \xi_{i,t}(l)$$

- The inclusive value of asset class l, $1 + \sum_{m=1}^{N} \delta_{i,t}(m,l)$, is related to the weight in the outside asset of asset class l

Market Clearing

- I investor countries, wealth $A_{i,t}$ across N countries and three asset classes
- Portfolio weight
$$w_{i,t}(n,l) = w_{i,t}(n|l)w_{i,t}(l)$$
- Outside asset investment $O_{i,t}$
$$A_{i,t} = \frac{O_{i,t}}{1 - \sum_{l=1}^{3}\sum_{n=1}^{N} w_{i,t}(n,l)}$$
- Market clearing
$$P_t(n,l)E_t(n)Q_t(n,l) = \sum_{i=1}^{I} \frac{O_{i,t}w_{i,t}(n,l;P_t,E_t)}{1 - \sum_{k=1}^{3}\sum_{m=1}^{N} w_{i,t}(m,k;P_t,E_t)}$$
- Short rate estimated separately, related to $\mathbf{z}_{i,t}$

Assumptions

- $O_{i,t}$ is exogenous for all investors (in USD)
- Characteristics $x_{i,t}(n)$ are exogenous
- Expected returns are determined through a statistical predictive regression

2.3. Data, instrument and estimation result

Data: CPIS Holding

- Coordinated Portfolio Investment Survey: bilateral holding of assets across countries, in short-term debt, long-term debt, and stocks, annual frequency
- FX reserve (foreign assets held by central banks): aggregated over all central banks (treat as one single investor)
- Supply of debt: those held by foreign investors
- Supply of equity: total market cap
  - Domestically held equity = total market cap - foreign holding
- Use BIS information on currency composition to separate local and foreign currency debt, only include local currency asset
- Tax haven issue: restate amounts outstanding from residency to nationality using the restatement matrix by Coppola et al (2021)

Data: Asset Price Data

- Short-term rate: 3-month interbank rate
- Long-term rate: 10-year benchmark government bond yields
- Stock returns and market-to-book equity

Data Limitations

- Disaggregate foreign reserve holding
- Only cover portfolio debt and equity, not others, especially fund shares and FDI
- Not adjust for currency hedging - depending on who the counterparty is

Characteristics

- Macro variables: log nominal GDP, log real GDP per capital, inflation
- Financial variables: equity volatility, sovereign debt rating
- Bilateral variables: export share, import share, distance
- Include a dummy for FX reserve investor and include time-series dummies

Identifying Assumptions

$$E\left[\varepsilon_{i,t}(n,l),\xi_{i,t}(l)|x_t,Q_t,O_t\right]=0$$

- This assumption does not hold by construction
- An instrument for expected return is required
- Implicit utilizing market clearing conditions

Instrument Construction: Within Asset Class

- Reduced-form regression and get the fitted value $\hat{\delta}_{i,t}(n,l)$

$$\log\left(\frac{w_{i,t}(n|l)}{w_{i,t}(0|l)}\right) = \Gamma_l' D_i(n) + v_l + \tau_{l,t} + \eta_{i,t}(n,l)$$

- Construct predicted value $\hat{w}_i(n,l)$ as

$$\hat{w}_i(n,l) = \frac{\hat{\delta}_i(n,l)}{1+\sum_{m=0}^{N}\hat{\delta}_i(m,l)}\hat{w}(l)$$

where $\hat{w}(l)$ is the mean portfolio weight on asset class l

- Predict supply using GDP and population only and get the fitted value $\hat{q}_t(n)$

$$q_t(n,l) = \theta_{1,l} + \ln GDP_t(n) + \theta_{2,l}\ln \text{Popu}_t(n) + \xi_{l.t} + e_t(n,l)$$

- Instrument for $\mu_{i,t}(n,l)$ is

$$IV_{i,t}(n,l) = \log\left(\sum_{j\neq i}\frac{O_{j,t}\hat{w}_j(n,l)}{1-\sum_l\sum_{n=1}^{N}\hat{w}_j(n,l)}\right) - (v_t(n) + \hat{q}_t(n))$$

Understanding the Instrument

- The instrument for expected return is essential the difference between predicted demand (excluding i) and predicted supply
- If a country n is large (high supply) and investors are distant (low demand), the expected return of the country's asset is high (low predicted price)
- Essentially, the IV is $p_t(n, I) + e_t(n) - v_t(n)$ constructed using the predicted demand and supply
- Size and distance are both exogenous, but the predicted price should have high correlation with the expected return

Instrument Construction: Across Asset Class

- Construct instrument for $\hat{w}_i(0|l)$ as

$$\hat{w}_i(0|l) = \frac{1}{1 + \sum_{m=0}^{N} \hat{\delta}_i(m, l)}$$

- Straightforward to run the across-asset-class regression

Demand Elasticity

The demand elasticity

$$1 - \frac{\partial \log \left(\sum_{i=1}^{I} A_{i,t} w_{i,t}(n,l) \right)}{\partial p_t(n,l)} = - \left(\frac{\partial p_t(n,l)}{\partial q_t(n,l)} \right)^{-1}$$

Needs to be solved numerically, see Appendix C of the paper for details

Estimation: Expected Returns

Table 3. Predictive Regressions

Variable	Exchange rate	Long-term debt	Equity
Log asset price		-0.74	-0.15
		(0.11)	(0.22)
Log real exchange rate	-0.27	-0.36	-0.54
	(0.07)	(0.07)	(0.28)
Constant		-0.07	0.25
		(0.02)	(0.20)
R^2	0.17	0.32	0.12
Observations	424	640	640

Log asset price is minus maturity times log yield for long-term debt and log market-to-book for equity. All models include country fixed effects. Robust standard errors clustered by year are reported in parentheses. The annual sample period is 2003 to 2020.

Estimation: Within Asset Class

Table 4. Estimated Demand within Asset Class

Variable	Short-term debt	Long-term debt	Equity
Expected return	14.33	4.52	10.33
	(2.32)	(0.51)	(0.79)
Log GDP	1.28	1.10	1.32
	(0.02)	(0.01)	(0.02)
Log GDP per capita	3.67	2.16	3.68
	(0.35)	(0.11)	(0.19)
Inflation	-23.49	-9.22	-16.56
	(4.22)	(1.79)	(1.88)
Volatility	-2.83	-0.52	-5.89
	(0.40)	(0.27)	(0.36)
Rating	-0.77	10.24	13.96
	(1.26)	(1.29)	(1.23)
Distance	-0.08	-0.18	-0.15
	(0.01)	(0.00)	(0.01)
Indicator variables:			
Domestic ownership	8.46	6.19	7.69
	(0.18)	(0.09)	(0.14)
Reserves	0.01	0.10	-2.83
	(0.19)	(0.10)	(0.14)
Other countries	0.78	0.77	-1.86
	(0.17)	(0.06)	(0.10)
Constant	-52.35	-34.78	-50.94
	(3.67)	(1.15)	(2.14)
F-statistic for weak IV	130	1,297	521
Observations	20,549	23,431	23,779

Expected returns are the predicted values from the predictive regressions in Table 3. The sovereign debt rating is a continuous measure equal to -1 times the ten-year default rate. All models include year fixed effects. Heteroskedasticity-robust standard errors are reported in parentheses. The critical value for a test of weak instruments at the 5 percent significance level is 16.38 (Stock and Yogo 2005). The annual sample period is 2003 to 2020.

Estimation: Across Asset Class

Table 5. Estimated Demand across Asset Classes

Variable	Symbol	Estimate
Log outside portfolio weight:		
Short-term debt	ρ_S	0.25
		(0.03)
Long-term debt	ρ_L	0.53
		(0.05)
Equity	ρ_E	0.49
		(0.04)
Indicator variables:		
Short-term debt	α_S	-1.21
		(0.19)
Long-term debt	α_L	0.73
		(0.18)
F-statistic for weak IV		802
Observations		1,352

Heteroskedasticity-robust standard errors are reported in parentheses. The annual sample period is 2003 to 2020.

Demand Elasticities

- Demand elasticity with respect to expected return: 14.33 for short-term debt, 4.52 for long-term debt and 10.33 for equity
- Elasticity across asset classes: 0.25 for short-term debt, 0.53 for long-term debt and 0.49 for equity
- Translate into aggregate price elasticity: 25.2 for short-term debt, 3.1 for long-term debt and 1.2 for equity
- Check the paper for a comparison with the literature

2.4. Applications

Exchange Rate and Asset Price Decomposition: Framework

As in KY (2019), we can write asset prices as a nonlinear function

$$\begin{bmatrix} e_t \\ p_t(2) \\ p_t(3) \end{bmatrix} = g(x_t, z_t, O_t, p_t(1), Q_t, \varepsilon_t, \xi_t)$$

- Macro variables $x_t, z_t, O_t, Q_t(3)$
- Short-term rates $p_t(1)$ and $Q_t(1)$
- Long-term debt quantities $Q_t(2)$
- FX reserves ε_t, ξ_t corresponding submatrix
- Latent demand ε_t, ξ_t corresponding submatrix

Exchange Rate and Asset Price Decomposition: Framework

- Recursively define counterfactual exchange rates
  - $e_{1,t+1}$, change macro variables from t to $t+1$ values
  - $e_{2,t+1}$, plus, change short-term rates from t to $t+1$ values
  - ...

$$var(e_{t+1}-e_t) = cov(e_{t+1}-e_t, e_{1,t+1}-e_t)$$

$$+cov(e_{t+1}-e_t, e_{2,t+1}-e_{1,t+1}) + cov(e_{t+1}-e_t, e_{3,t+1}-e_{2,t+1}) +$$

$$cov(e_{t+1}-e_t, e_{4,t+1}-e_{3,t+1}) + cov(e_{t+1}-e_t, e_{5,t+1}-e_{4,t+1})$$

Decomposition Results

TABLE 7. VARIANCE DECOMPOSITION OF EXCHANGE RATES AND ASSET PRICES

Change in	Exchange rate	Short-term rate	Long-term yield	Market-to-book equity
Portfolio flows	0.01		0.58	0.20
	(0.05)		(0.19)	(0.07)
Macro variables	0.16	0.14	0.41	0.19
	(0.06)	(0.06)	(0.09)	(0.03)
Latent demand	0.83	0.86	0.01	0.61
	(0.07)	(0.06)	(0.26)	(0.07)
Reserves	0.10		0.02	-0.01
	(0.02)		(0.03)	(0.01)
North America	0.32	0.45	-0.23	0.15
	(0.09)	(0.14)	(0.20)	(0.08)
Europe	0.22	0.17	0.16	0.26
	(0.04)	(0.06)	(0.06)	(0.04)
Pacific	0.22	0.03	-0.02	0.09
	(0.06)	(0.02)	(0.02)	(0.02)
Emerging markets	-0.05	0.21	0.08	0.12
	(0.02)	(0.06)	(0.07)	(0.05)
Other countries	0.02		0.00	0.00
	(0.01)		(0.01)	(0.00)
Observations	399	416	603	603

Heteroskedasticity-robust standard errors are reported in parentheses. The observations are value-weighted by the market weights within year and asset class. The annual sample period is 2003 to 2020.

Exchange Rate Disconnect?

- Exchange rates are determined by macro, financial, and debt supply characteristics of **all** countries, not just bilateral
- The highly nonlinear function implied by holdings

Convenience Yield on US Assets

- Include an US issuer FE in the regressor
- Counterfactual: set it zero, what is the counterfactual bond yield? - measuring the convenience yield

Table 10. Convenience Yields on US Assets

	Foreign short-term debt		US long-term debt		US equity	
Investor	Exchange rate	Expected return	Yield	Expected return	Market-to-book	Expected return
Total	5.36	-1.45	0.76	2.81	-3.37	0.50
	(0.58)	(0.16)	(0.10)	(0.36)	(0.40)	(0.06)
Reserves	3.49	-0.94	0.28	1.02	-0.07	0.01
	(0.44)	(0.12)	(0.03)	(0.13)	(0.01)	(0.00)
North America	0.07	-0.02	0.01	0.05	-0.37	0.05
	(0.01)	(0.00)	(0.00)	(0.00)	(0.04)	(0.01)
Europe	0.87	-0.24	0.23	0.85	-1.77	0.26
	(0.09)	(0.03)	(0.03)	(0.12)	(0.20)	(0.03)
Pacific	0.53	-0.14	0.21	0.78	-0.88	0.13
	(0.07)	(0.02)	(0.03)	(0.11)	(0.10)	(0.02)
Emerging markets	0.14	-0.04	0.01	0.05	-0.16	0.02
	(0.02)	(0.00)	(0.00)	(0.01)	(0.04)	(0.01)
Other countries	0.26	-0.07	0.01	0.05	-0.12	0.02
	(0.03)	(0.01)	(0.00)	(0.01)	(0.02)	(0.00)

This table reports the time-series mean of the counterfactual changes in exchange rates and asset prices in the absence of special demand for US assets, reported in annual percentage points. Special demand is estimated as the cross-sectional mean of latent demand for US assets by year and asset class, excluding the US investors' latent demand. Expected returns are the predicted values from the predictive regressions in Table 3. Heteroskedasticity-robust standard errors are reported in parentheses. The annual sample period is 2003 to 2020.

What's New in This Paper: Summary

- Expected return modeled through a statistical predictive relation and enter asset demand function explicitly
- Two-tier asset demand: within- and across-asset-class
- A new approach to construct the instrument
- An example of using aggregate data in demand system estimation

Related Literature

- Apply these approaches to a different asset market
- **The inelastic market hypothesis and the granular instrumental variable**
- **Methodological improvement**