# Discussion: The Value of Arbitrage

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# This Paper

- Research question: What is the social value of arbitrage?
  - Arbitrageurs that close the arbitrage gap play fundamental roles in modern finance
  - Little is explored from an welfare perspective
  - A related (complementary) paper: Opp and van Binsbergen (2019, JF)

# This Paper

- Research question: What is the social value of arbitrage?
  - Arbitrageurs that close the arbitrage gap play fundamental roles in modern finance
  - Little is explored from an welfare perspective
  - A related (complementary) paper: Opp and van Binsbergen (2019, JF)
- A framework and a (**very cool**) sufficient statistic result
- Empirical applications in CIP deviation and dual-listed firms

# Insights from the Simple Model: Framework

- Two investors, two assets with the same cash flow $d_1$

$$\max_{q_0^i} u_i(c_0^i) + \beta_i u_i(c_1^i)$$

$$s.t. : c_0^i = n_0^i - p^i(q_0^i - q_{-1}^i), c_1^i = n_1^i + d_1 q_0^i$$

Optimality condition implies $p^i = \frac{\beta_i u_i'(c_1^i)}{u_i'(c_0^i)} d_1$

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- Arbitraguers

$$c_0^\alpha = -p^A q_0^{\alpha A} - p^B q_p^{\alpha B} = (p^B - p^A)m$$

  where $m = q_0^{\alpha A}$.

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- Arbitrage equilibrium
  - An arbitrage equilibrium is parameterized by $m$
  - Investors maximize utility, arbitrageurs receive zero consumption in period 1 and both asset markets clear
  - **Arbitraguers do NOT optimally choose** $m$, allowing for rich flexibility

# Insights from the Simple Model: Main Results

- Marginal value of arbitrage

$$\frac{dW(m)}{dm} = p^B(m) - p^A(m)$$

  - Distributive effects: arbitrageurs buying asset A raises the price of A and lowers the price of B - zero in aggregate
  - Pecuniary effects: gains of the arbitrageur

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  - Pecuniary effects: gains of the arbitrageur

- Total value of arbitrage

$$W(m^*) - W(m_0) = \int_{m_0}^{m^*} (p^B(\tilde{m}) - p^A(\tilde{m}))d\tilde{m}$$

$$p^B(\tilde{m}) - p^A(\tilde{m}) = \underbrace{p^B(m_0) - p^A(m_0)}_{\text{observable}} + \int_{m_0}^{\tilde{m}} \left( \underbrace{\frac{dp^B(\hat{m})}{dm} - \frac{dp^A(\hat{m})}{dm}}_{\text{price impact}} \right) d\hat{m}$$

  - $m_0$ is the status quo, $m^*$ is the arbitrage trade that closes the gap

# Generalization

- In the paper, the authors extend the framework for multi-agent, multi-asset, dynamic stochastic environment
- Including some important cases
  - Restrictions on future arbitrage trade (Shleifer and Vishny, 1997)

# Empirical Applications

- CIP deviation
  - Measure CIP price impact under a series of assumptions, all of which bias welfare gains upward
  - A small welfare gain of closing the gap
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  - Measure CIP price impact under a series of assumptions, all of which bias welfare gains upward
  - A small welfare gain of closing the gap
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- Dual-listed firms
  - The welfare gain of arbitrage differs substantially across different firms
  - Large for Shell/Royal Dutch, and smaller for Smithkline Beecham and Rio Tinto

# Comments

- No arbitrage
- Micro vs. macro price impact
- Small comments on the CIP application
- The welfare notion

# No Arbitrage

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  - To my understanding, the methodology of this paper is applicable to the broad definition of no arbitrage, though the applications are restricted to the narrow one
  - For a given model, arbitrage opportunities may induce arbitrageurs to trade for "alphas", and thus we can measure price impact

The authors wrote, "While our welfare assessments are exact in models in which there are strict arbitrage opportunities, one may conjecture that our results remain approximately valid for quasi-arbitrages. Extending our approach to those situations is a fascinating topic for further research. "

# Methodology

- The novelty of the paper's methodology is to define an arbitrage equilibrium as a function of arbitrageurs' trade size
  - Allow for very flexible microfoundations of arbitrage
  - Very cool: **Price impact** and arbitrage gap are sufficient statistic

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  - Allow for very flexible microfoundations of arbitrage
  - Very cool: **Price impact** and arbitrage gap are sufficient statistic
  - It well captures the effect of arbitrageurs on resource allocation

$$p^i = \frac{\beta_i u_i'(c_1^i)}{u_i'(c_0^i)} d_1$$

# Micro vs. Macro Price Impact

- Based on the theoretical framework, we need to know the "macro" price impact, but in applications the authors use "micro" price impact

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  - Improved resource allocation
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- "Micro" price impact: the market liquidity, i.e., security price change induced by trading
- "Macro" price impact can be richer, for example:
  - Improved resource allocation
  - Enhanced risk sharing
  - Information provision
- It will be very cool if the authors can propose a method to identify the "macro" price impact
- One thought: Opp and van Binsbergen (2019) provide a framework to measure the resource allocation effect of eliminating alphas, which is complementary to the welfare framework provided by this paper

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- In several cases, there may be "extrapolation" problems, e.g., heterogeneous arbitraguers
    - European banks report snapshot quarter-end leverage ratio, while US banks use quarter aerage

# The Welfare Notion

- The authors are aware of the limitation of the approach: not considering the potential cost of closing the arbitrage gap
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- The authors are aware of the limitation of the approach: not considering the potential cost of closing the arbitrage gap
  - For example, optimal regulation or marginal requirement
- A more comprehensive welfare analysis requires specifying the microfoundation
- However, it will be helpful to show, in some leading examples, the relative welfare benefit and the ignored welfare cost

# Conclusion

- A great paper, an important big question, novel methodology and easy-to-follow illustration, well-executed empirical applications, should be on everyone's reading list

# Conclusion

- A great paper, an important big question, novel methodology and easy-to-follow illustration, well-executed empirical applications, should be on everyone's reading list
- Some discussions on the applicability of the methodology (beyond strict arbitrage opportunities)
- Discuss more about the micro vs. macro price impact
  - How to get "macro" price impact is challenging but can be super cool
  - Some comments on the CIP deviation application about heterogeneous arbitrageurs
- Welfare notion: more examples to clarify the relative importance of welfare gain and potential welfare loss in specific leading examples
  - Regulation, risk management, marginal requirement, etc