Discussion: Production-based Stochastic Discount Factors

Authors: Frederico Belo and Xinwei Li

Xiang Fang

The University of Hong Kong

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  - From the asset return space
  - From consumption-based models
  - **From the production-based models**
    - Belo (2010): technology flexibility across states, infer SDFs from output and price data
    - This paper: infer from investment Euler equation

This Paper in Three Steps

- Establish a link between SDF and investment rates
- Estimate the model to construct recovered SDF
- Price the cross-section of equities, implications on the term structure

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$$K_{t+1} = (1-\delta)K_t + I_t, \Phi(I_t, K_t, \mathcal{E}_t) = \frac{\kappa}{\eta+1}\left(\frac{I_t}{K_t}\right)^{\eta+1}\Pi_t$$

- Optimal investment problem $X_t = (\mathcal{E}_t, K_t)$

$$V(X_t) = \max_{I_t} D_t + E_t\left[M_{t+1}V(X_{t+1})\right]$$

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- Investment Euler equation

$$E_t\left[M_{t+1}(X_{t+1})R^I_{t+1}(X_{t+1})\right] = 1$$

where

$$R^I_{t+1} = \frac{\mathcal{E}_{t+1}\left(1 + \kappa\frac{\eta}{\eta+1}IK^{\eta+1}_{t+1} + (1-\delta)\kappa IK^{\eta}_{t+1}\right)}{\mathcal{E}_t\kappa IK^{\eta}_t}$$

Parameterization

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- The dynamic system

$$\Delta\varepsilon_{t+1} = \mu_\varepsilon + s_t + \sigma_t e_{\varepsilon,t+1}$$

$$s_{t+1} = \mu_s + \rho_s s_t + \varphi_s \sigma_t e_{s,t+1}$$

$$\sigma^2_{t+1} = \mu_\sigma + \rho_\sigma \sigma^2_t + \sigma_\sigma e_{\sigma,t+1}$$

- SDF specification

$$m_{t+1} = -\mu_m - \rho^m_s s_t - \rho^m_\sigma \sigma^2_t - \lambda^m_\varepsilon \sigma_t e_{\varepsilon,t+1} - \lambda^m_s \sigma_t e_{s,t+1} - \lambda^m_\sigma \sigma_\sigma e_{\sigma,t+1}$$

- Conjecture the solution and solve for undetermined coefficients using investment Euler equation

$$ik_t = \alpha + \beta s_t + \phi \sigma^2_t$$

- Risk-free rate $r_{f,t}$

$$r_{f,t} = a + b s_t + c\sigma^2_t$$

These two equations exactly back out s_t and σ_t from ik_t, $r_{f,t}$

Estimation

- Approach
  - Given parameters, use ik_t and $r_{f,t}$ to exactly back out s_t and σ_t
  - Plug the recovered s_t and σ_t into the dynamic system and SDF, calculate moments of interest
  - Minimize distance between data moments and model implied moments

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- In this model, investment is driven by expected profitability
- Empirically, investment and profitability are driven by different sources of risks (but related)
- Can the model address this issue?
  - Allowing for a certain degree of separation between investment and profitability will be useful in bringing the model closer to the data

Comment #2: The Affine-Linear Specification

- The paper essentially takes an affine-linear approach
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- The paper essentially takes an affine-linear approach
  - Expected profitability is a linear AR(1) process
  - SDF loads on shocks linearly
  - Investment ratio is affine linear in states
- Simple and tractable, but remains to be seen whether the approximation is accurate
  - In a full-fledged model, time-varying risk premium can have significant nonlinear effects on expected profitability, investment and SDF
  - Useful if the authors can assess the validity of affine-linear specification through example full-fledged models

Comment #3: Richer Investment Dynamics

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- Investment has richer dynamics: lumpy investment, asymmetry between investing and divesting, etc
- Could be useful to incorporate such richer dynamics to show the robustness of results

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  - Invert the system and back out latent variables
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  - Observables are subject to measurement error, not necessarily a good idea to do a direct mapping
  - Alternative: estimate the dynamic system through Kalman filter
  - Stochastic volatility: Bayesian approach (Primiceri, 2005; Kim, Shephard and Chib, 1998)
  - Not necessary to use equity/risk-free rate in the recovery
  - At least useful to show the robustness of results

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- Clarify the value-added of this investment approach compared to the production approach (Belo, 2010; Jermann, 2013, etc)

Conclusion

- A very nice paper, simple and elegant idea, nice implementation
- Should be on the reading list of anyone interested in production based asset pricing and macro finance
- Comments
  - Investment and profitability
  - The economic structure
  - Estimation approach
  - Clarify the contribution